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"POWER SPECTRAL CHARACTERIZATION OF INTERFACIAL
FLUCTUATIONS IN CO-CURRENT OIL-WATER FLOW"

BY

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The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies for acceptance, a thesis entitled "Power Spectral Characterization of Interfacial Fluctuations in Co-Current Oil-Water Flow", submitted by Theodore D. Strashok in partial fulfillment of the requirements for the degree of Master of Science in Chemical Engineering.

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ABSTRACT

Experiments conducted in this investigation were aimed at characterizing the interfacial structure developed when oil and water flowed co-currently in stratified flow. Experiments were conducted in a horizontal rectangular conduit having an aspect ratio of 7.95:1. The description of the interface was accomplished utilizing a photometric technique which produced a signal that enabled the mean height of the oil along with the root-mean-square surface displacements, which gave an indication of the size of the waves present, to be measured. A wide range of flow rates was covered in order that the salient features of each flow regime might be discovered.

A power spectral analysis was conducted at these flow rates to describe the distribution of frequencies present. The maxima of these spectral density functions indicate the dominant frequency present at the flow rate in question. In the small two-dimensional wave regime, a maximum was noted at values of frequency within the 5 cps to 18 cps range. In the three-dimensional wave regime a maximum was also noted, but the spectral curve was much flatter indicating that no one frequency was dominant. In the large two-dimensional wave regime, two or three maxima were observed. At least one of these indicated a monotonic increase in the dominant

frequency with water Reynolds' number, the oil Reynolds' number remaining constant. These higher frequency disturbances appear to be caused by turbulent pressure fluctuations. In this regime a lower frequency, higher energy dominant frequency also developed as the water flow rate increased. These larger waves appeared to travel at a lower rate than the mean flow indicating perhaps, that they were due to some instability in the mean flow. Two different mechanisms of wave inception thus appeared to be present within the range of this investigation.

The insitu water to oil ratios in laminar-laminar flow agreed exactly with earlier theoretical predictions. One of the most important conclusions to be drawn from this work is that a photometric technique is suitable for obtaining data useful in characterizing the interfacial structure of two immiscible liquids flowing co-currently in a rectangular conduit. This is based on the reasonable agreement found between the actually measured total power and that obtained by integration of the power spectral density function.

I. INTRODUCTION

Experiments were conducted in a 35.7 foot horizontal rectangular conduit at sufficiently high flow rates to produce wave-like interfacial disturbances. These fluctuations were observed on the interface between two liquids, oil and water, travelling in co-current stratified flow. An attempt at characterizing the interface was made by means of a power spectral analysis of the waves along with the determination of the position of the interface at various flow rates.

A. STRATIFIED FLOW OF TWO IMMISCIBLE LIQUIDS

The flow of two phases in a horizontal conduit is encountered in situations such as pipeline transportation of oil, tubular reactors, and film coolers⁽¹⁶⁾. The most prevalent type of flow pattern in most situations is that of stratified flow. Stratified flow, which is but one of many types of flow which may occur in a two phase flow system, can be broken down into five characteristic regimes which are described in the next section. Govier and Omer⁽¹³⁾ have described other regions of two phase flow established by air and water travelling concurrently in a one inch pipe. Stratified flow occurs at low flow rates of both phases. As the gas flow rate increases, waves develop, the flow still being stratified. Further increase in the gas volume input results in "annular flow" with the gas forming the core. An initially low gas

flow rate and high liquid flow rate results in "bubble flow" with bubbles of gas travelling along the top of the pipe. A subsequent increase in gas flow rate establishes "plug flow" wherein large gas bubbles traverse the upper part of the pipe. A further increase results in "slug flow" in which slugs of liquid occasionally pass through the pipe.

A viscous fluid, such as an oil, when being transported by pipeline requires much power due to the large viscous frictional dissipation present in the thin layer of fluid next to the conduit wall where the velocity gradient is steepest. Introduction of a less viscous fluid, such as water, into this region would result in decreased frictional losses. Concentric flow of oil in water has been shown to be the most effective means of reducing the required pressure gradient⁽³¹⁾. A slight density difference, though, causes the two phases to become stratified. A means of correlating the decrease in pressure gradient upon the addition of a less viscous fluid is that of the pressure gradient reduction factor. It is defined as the ratio of the pressure gradient for the flow of the more viscous fluid at a given flow rate to the pressure gradient for the two phase system with the more viscous fluid flowing at the same rate. It has been shown that even stratified flow results in a pressure gradient reduction factor which is greater than unity for proper water-oil input ratios⁽⁵⁾.

Gazley⁽¹²⁾ noted that at the point where the interface between air and water in stratified flow became wavy, an increase in slope in

the curve representing the two phase pressure gradient and superficial air velocity resulted. This indicates a relatively higher pressure loss with waves present. This fact, coupled with the almost inevitable existence of stratified flow in two phase co-current flow, makes the study of waves formed in stratified flow an important field of investigation.

B. CHARACTERIZATION OF INTERFACIAL STRUCTURE

Previous work ⁽³⁾ with two liquids flowing in stratified flow in a rectangular conduit indicated the presence of the following five flow regimes:

- (i) smooth - both phases in laminar flow results in a completely smooth interface
- (ii) small two-dimensional waves - long crested, small amplitude waves of small wavelength travelling in the flow direction
- (iii) large two-dimensional waves - crests extending over the complete width of the conduit with smaller waves superimposed upon them
- (iv) roll waves - sharp, irregular crests of variable wavelength with small waves still superimposed upon them
- (v) three dimensional waves - short crests having a three dimensional character with the crests approximately equal to the wavelength

Similar interfacial disturbance patterns have been observed by Hanratty and Engen ⁽¹⁴⁾ in two phase air-water flow. However, the roll waves they described differed from those observed here, in that for air-water flow, small droplets of water seemed to have been picked up

and carried along with the air stream. In oil-water flow this phenomenon did not occur, probably due to the flow rates not being high enough to allow this type of dispersed flow.

These experimental flow regimes, which agreed with those observed in this investigation, are summarized in Figure 1. Photographs of these wave structures appear elsewhere⁽⁵⁾.

The characterization of water waves by a statistical approach represented by a stationary Gaussian process appears to yield quite realistic results, especially for lower frequency waves. Considerable work utilizing this method has been done by oceanographers such as Pierson and Marks⁽²⁹⁾. Lilleleht⁽²³⁾ adopted this statistical approach for an air-water system flowing in a rectangular conduit. From the analysis of wave records he determined that the time distribution of instantaneous liquid film heights fitted a Gaussian model over most of the range of surface elevations. Deviations at higher wave heights were attributed to the effect of the channel bottom. In addition to his wave record analysis, Lilleleht made measurements of the power spectral density function for various flow conditions in the frequency range 0.2 to 60 cps. Using these measurements he was able to calculate the number of times the surface crossed the mean liquid level in unit time $N(\bar{h})$ from Equation 28, derived for Gaussian noise. Good agreement was found between the calculated values of $N(\bar{h})$ and those obtained from wave records.

Various non-statistical approaches for describing the liquid surface for an air-water system are aptly outlined by Lilleleht⁽²³⁾ and Cohen⁽⁷⁾.

For the liquid-liquid system a non-statistical approach was used by Charles⁽³⁾ in describing the surface structure. He measured the wave velocities of the regular waves by simply timing the movement of a particular wave over a distance of three feet. Wavelengths and amplitudes of these regular waves were estimated utilizing a stereophotogrammetric technique⁽⁴⁾. Velocity profiles were predicted theoretically and measured experimentally for laminar-laminar flow.

Before turbulent-laminar or turbulent-turbulent cases can be treated, a means must be found to represent the velocity distribution in the turbulent phase in a rectangular conduit by something similar to the universal velocity distribution for single phase flow in circular conduits or between infinite parallel plates. Sinha⁽³²⁾ has measured single phase velocity profiles for turbulent flow of water in the rectangular conduit also used for this investigation. Two phase turbulent velocity distributions have not as yet been determined for the system in question.

Two liquids travelling in co-current, stratified flow are thus capable of generating waves at their interface. These waves result in a higher surface shear stress and a correspondingly higher pressure loss than if the surface is smooth. This in turn results in a decrease

in the pressure gradient reduction factor. Thus a better knowledge of the character of these waves along with aiding in the understanding of their cause and growth would help in predicting increased pressure losses due to the presence of the waves.

C. GENERATION OF WAVES

Many theories have been presented over the years on the generation and behavior of waves. If the fluid under consideration could be classified as inviscid, the problem would merely be one of mathematics. For real fluids, though, the resources of mathematics have not as yet yielded a complete solution. When a rigorous mathematical solution is not possible, a model must be developed. This model should be based on a combination of theory and experiment, utilizing appropriate simplifying assumptions. A number of such models have been advanced to date.

Many of the early theories and models concerning waves on open water have been presented in Lamb⁽¹⁷⁾ - notably those of Rayleigh⁽¹⁸⁾, Scott Russell⁽¹⁹⁾, and Kelvin-Helmholtz⁽²⁰⁾. Scott Russell presented primarily, observations of wave structure at various wind velocities. He noted that at a wind velocity of about one mile per hour low amplitude capillary waves formed which were incapable of travelling spontaneously to any considerable distance, except when under the continued action of the original disturbing force. At a wind velocity of about two miles per hour small gravity waves arose uniformly over the whole water surface, with the capillary waves superimposed in the sheltered hollows between them.

Much work on waves was carried out by Rayleigh. One of his contributions was the assumption that the deviation of any

particle of the fluid from the state of uniform flow is resisted by a force proportional to the relative velocity of the waves to the water. Such an assumption introduced the concept of small dissipative forces to the assumptions inherent in the perfect fluid theory.

The Kelvin-Helmholtz theory was among the first to propose a mathematical model describing the minimum velocity at wave inception. This theory is based on a small perturbation imposed upon the equations of motion of the aforementioned fluids with the air moving over a deep stagnant fluid. The model assumes uniform flow in each phase, a discontinuous velocity distribution at the interface and negligible viscosity. It is further assumed that the motion is controlled by gravity and surface tension alone. These assumptions result in the motion of the interface being described by a velocity potential. The critical wind velocity, that is the velocity for which the disturbances will neither grow nor decay, is predicted to be about fifteen miles per hour, whereas in nature waves are observed at wind velocities very much below this value.

Turbulence has been taken into account by other, more recent workers. One major theory proposed utilizes the contribution of air turbulence to wave generation. This theory, due to Phillips⁽²⁸⁾, is concerned with the resonance-effect of pressure fluctuations in the turbulent air stream on an initially calm water surface. Another theory

along the same lines is that of Eckart⁽⁹⁾. Many of his assumptions, though, result in an unrealistic approach, and in fact, the heights of predicted waves are less than those observed.

Jeffreys⁽¹⁵⁾, Miles⁽²⁵⁾, and Benjamin⁽¹⁾ have proposed theories utilizing the instability of an interface between a viscous fluid and air as a criterion for wave generation. These workers took turbulence into account by illustrating its effect on the mean shear flow.

Jeffreys, in his model, introduced a semi-empirical concept of "sheltering". He concentrated his efforts on the pressure difference between the windward and leeward sides of the waves caused by separation of the air stream on the leeward side. When the energy produced by this pressure difference exceeded the viscous dissipation, waves once formed would continue to grow in amplitude. In order to have his model agree with observed conditions, he introduced a sheltering coefficient to express the magnitude of the pressure difference. This coefficient, calculated from experimental observation of the critical wind velocity, had a value of approximately 0.30.

Miles^(25, 26), in extending Jeffreys' work, showed theoretically that a larger value of the sheltering coefficient may be obtained for a moving wave system. Miles showed that these large values occur when a critical point - where the velocity of the gas equals the wave velocity - exists away from the disturbed interface. No flow separation

is required in this case, as a significant pressure component in phase with the wave slope arises due to a phase change in the disturbed gas velocity across the friction layer surrounding the critical point. Miles concludes that his theory is in line with that of Phillips.

Benjamin⁽¹⁾ and Miles⁽²⁷⁾ furthered Miles' initial work by showing that viscous stresses in the vicinity of the wavy surface can also cause a phase change in the disturbed gas flow. This, along with the phase change in the critical layer, play complementary roles, the former being dominant for short waves. Benjamin, in his investigation, observed the flow of air over thin layers of highly viscous syrup and of water. In the case of the syrup, long crested waves appeared, while with water, the long crested waves were preceded by an apparently random distribution of waves. Benjamin thus concluded that waves can be initiated by two different mechanisms. He proposed that the random distribution of waves was caused by turbulent pressure fluctuations, and that the long crested waves appeared to originate in the instability of the mean flow of air and water adjacent to the interface. In turbulent flow, turbulent fluctuations of one fluid will be transmitted, to some degree, to the second fluid, depending on the properties of the fluids involved.

A major assumption in Benjamin's theory of instability involves the existence of a laminar sublayer, with its uniform

velocity profile. According to him, the integral relationship for the pressure fluctuation amplitude is insensitive to the contribution from the viscous layer in the vicinity of the interface, that is, the pressure does not vary through the viscous layer. If the densities and viscosities of two fluids were similar, Reynolds stresses would predominate and there would be a decrease in thickness of the viscous sublayer at the boundary. Two fluids such as air and water have a large difference in density and viscosity, and in this case the laminar sublayer is similar to that in flow over a solid boundary.

Benjamin's theory is also concerned with the influence of a disturbed interface on the air flow. That is, if the random disturbances are already present, what effect will they have on the air flow? If the surface roughness is so minute that the viscous sublayer exceeds it in thickness, the moving interface will have only an effect of secondary importance on the structure of the air flow, and the mobility of the boundary becomes a factor of secondary importance to the structure of the air flow. This he illustrated with his syrup, water, and air experiments.

This theory presented by Benjamin seems to agree with work done by Cohen⁽⁷⁾, Lilleleht⁽²³⁾, Charles and Lilleleht⁽⁶⁾, and with the general observations noted in this investigation. Cohen, in his theoretical and experimental investigation of the interaction between turbulent air and a flowing liquid film, noted that the first disturbances

to form had the appearance of long crested waves. These occurred when the energy transferred from the gas to the liquid exceeded the energy dissipated in the film. A relationship for energy dissipation, and for energy transfer based on the theories of Jeffreys and Miles-Benjamin was developed. Cohen found that the Miles-Benjamin theory predicted air flow rates, wave velocities, and wavelengths which were in agreement with observed values at wave inception for both the two- and three-dimensional waves. Phillips resonance mechanism did not appear applicable to the experimental results reported, since the turbulent fluctuations were convected downstream with a velocity three to four times greater than the wave velocity.

Charles and Lilleleht⁽⁶⁾, however, observed that at least two basically different mechanisms may be responsible for the generation of waves at liquid-liquid interfaces. They noted that Phillips' resonance mechanism appeared to be responsible for the first disturbances, while the larger waves appeared due to some mean flow instability.

As Lighthill⁽²²⁾ points out, the pressure fluctuations due to turbulence in an air flow are much weaker than those estimated by Phillips. The pressure fluctuations in a liquid, though, are more easily transmitted. This, coupled with the lower density and viscosity differences which result in a smaller, constant pressure laminar layer, indicate this mechanism of wave generation to be important with regard to the inception of the first small waves in the two-liquid co-current stratified flow system.

Two different theoretical approaches to the problem of wave generation are thus possible - the first concerned with the generation of waves by turbulent pressure fluctuations, and the second based on an instability mechanism in the mean flow. The theory which would apply in a given naturally occurring or experimental system depends on the nature of the fluids involved and the flow condition at which the waves are observed.

D. SCOPE OF THE INVESTIGATION

Characterization of the interfacial structure developed between oil and water was carried out using both statistical and non-statistical arguments as the descriptive parameters. This endeavor was conducted as part of a continuing investigation into the various aspects of two-phase, co-current, stratified flow of oil and water in a horizontal rectangular conduit. Previous work with the conduit has been concerned with quantitative measurements of the effect of the addition of a less viscous phase on the pressure gradient of the more viscous oil phase along with velocity distributions in single phase flows. Some semi-quantitative work has also been done stressing the laminar-turbulent transitions within the individual phases which appeared to determine the structure of the interface.

This investigation involved the description of the interface over various flow rates within the range where previous work had indicated the presence of a disturbed interface. The range covered was limited by the capacities of the experimental equipment and was within the boundaries outlined by Charles⁽³⁾.

To obtain a quantitative description of the interfacial disturbances, a photometric technique for measuring the interfacial position at one point in the conduit as a function of time was

utilized. The resulting fluctuating signal was analyzed in two ways. A power spectral density function was determined which described the distribution of frequencies over the flow map covered. The time average position of the interface at that point was also measured. This mean height could be an important parameter, particularly with the shallow layers encountered in the conduit.

The power spectral measurements were obtained in an effort to describe the distribution of frequencies of the waves present at any one flow rate. An attempt was also made at uncovering the effects of the controllable flow variables, such as flow rate, on the maxima obtained in the spectra - both with respect to the total energy and the frequency of the maxima. From the total power present under the spectral density curve, the root-mean-square surface variation could be obtained which would give some indication of the relative sizes of the waves present at each flow rate.

These measurements were obtained over a wide range of flow conditions in order that a general description of the interface over the entire flow map would evolve.

II. THEORY

The often random nature of waves formed on an interface between two flowing liquids makes it impossible to represent the surface elevation with any simple closed mathematical expression. The surface structure and many of its average properties can be represented statistically using parameters such as the average position of the disturbed surface, in this case the thickness of the oil layer $\bar{h}$, and the deviation of the surface elevation h' from the mean.

An extremely useful method of obtaining many of the statistical properties of the system is through a power spectral analysis. This type of analysis has previously been applied to that field of fluid flow relating to the concept of turbulence⁽³⁴⁾.

A. POWER SPECTRAL ANALYSIS IN GENERAL

Given a random signal $X(t)$, the covariance at lag τ is given by⁽²⁾

$$C(\tau) = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} X(t) X(t+\tau) dt \quad (1)$$

The power spectral density is defined as the Fourier transform of this autocovariance function at zero lag.

$$C(\tau) = \int_{-\infty}^{\infty} P(f) e^{i 2\pi f \tau} df \quad (2)$$

where

$$P(f) = \lim_{T \rightarrow \infty} \frac{1}{T} \left| \int_{-T/2}^{T/2} X(t) e^{-i 2\pi f t} dt \right|^2 \quad (3)$$

and

$$X(f) = \int_{-\infty}^{\infty} X(t) e^{-i 2\pi f t} dt \quad (4)$$

$P(f) df$ represents the contribution to the total variance from frequencies between f and $(f+df)$. The word "power" is derived from the application of this type of analysis to electric signals. If $X(t)$ is considered as a voltage across a pure resistance of one ohm, the long-time average power dissipated in the resistance will be strictly proportional to the variance of $X(t)$.

The autocovariance and the power spectrum are even functions of their respective arguments. Thus the relationship between them may be expressed as the two-sided cosine transform

$$C(\tau) = \int_{-\infty}^{\infty} P(f) \cos(2\pi f \tau) df \quad (5)$$

or more simply as the one-sided cosine transform

$$C(\tau) = 2 \int_0^{\infty} P(f) \cos(2\pi f \tau) df \quad (6)$$

From Equation (1)

$$C(\tau) = \text{ave} [X(t) X(t+\tau)] \quad (7)$$

Setting $\tau = 0$, the variance or total power present is given by

$$\text{var} [X(t)] = \int_0^{\infty} 2 P(f) df \quad (8)$$

In this way, the power spectrum is represented over all frequencies.

Sometimes the power spectrum is referred to in terms of a power spectral density $2 P(f)$ over positive frequencies only.

There are two general methods of obtaining a power spectral density⁽³⁵⁾

- (1) digitally by sampling periodically over some time interval
- (2) utilizing an analog technique, by sampling continuously, with respect to time, over some frequency interval

This latter method, the one used in the present investigation, involves calculating the spectrum directly at those frequencies of interest. In this technique the signal is passed through a narrow-band filter after which the average output power within the pass-band frequencies is measured. As illustrated in Figure 2, the true original spectrum $P(f)$ is modified by the transfer function of the filter $Y(f, f_M)$ to yield the output spectrum $P_o(f_M)$. Expressed mathematically this operation yields

$$P_a(f_M) = \int_{-\infty}^{\infty} |Y(f, f_M)|^2 P(f) df \quad (9)$$

where $P_a(f_M)$ has been determined by averaging the output power $P_o(f, f_M)$ over a long time

$$P_o(f, f_M) = \left| Y(f, f_M) \right|^2 P(f) \quad (10)$$

$Y(f, f_M)$ is the transfer function of the filter with a narrow pass-band around the frequency f_M . Equation (9) can be expressed as a one-sided definition of the power spectrum taking advantage of the fact that $P_a(f_M)$ and $P(f)$ are even functions of their respective arguments. The function $\left| Y(f, f_M) \right|^2$ has one of the necessary properties of a physically true transfer function in that it is an even function of both f and f_M . The purpose of using $\left| Y(f, f_M) \right|^2$ as the modifying function for the true power spectrum $P(f)$ is proven elsewhere⁽²⁾. Retaining the power spectrum as a two-sided function, it may be written in the more convenient one-sided form as

$$2 P_a(f_M) = \int_0^{\infty} H(f, f_M) 2 P(f) df \quad (11)$$

where

$$H(f, f_M) = 2 \left| Y(f, f_M) \right|^2 \quad (12)$$

Equation (9) is the equation which must be solved to obtain the true power spectrum $P(f)$ knowing $P_a(f_M)$ from experimental data and $Y(f, f_M)$.

If $X'(t)$ is assumed to be a perturbation voltage fluctuating about some mean voltage $\overline{X}$, that is

$$X'(t) = X(t) - \overline{X} \quad (13)$$

then the root-mean-square perturbation voltage is

$$X'_r = \left[\lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T (X(t) - \overline{X})^2 dt \right]^{1/2} \quad (14)$$

Comparing Equations (3) and (4) it can be seen that

$$2 P_a(f) = 2 \lim_{T \rightarrow \infty} \frac{|X'(f)|^2}{T} \quad (15)$$

recalling from Equation (8) that $2 P(f) df$ is the amount of power between f and $(f + df)$ in the one-sided true power spectrum.

Obtaining a root-mean-square perturbation voltage, at any particular frequency, averaged over a sufficiently large time interval, results in the positioning of the signal function in image space without the need of taking its Fourier transform. From Equation (14) it can be seen that the root-mean-square perturbation voltage at the frequency of interest is

$$\left[X'_r(f) \right]^2 = \lim_{T \rightarrow \infty} \frac{|X'(f)|^2}{T} \quad (16)$$

where $|X'(f)|^2$ is the total power of the perturbation signal $X'(t)$ integrated over time T at frequency f . Thus from Equation (15) the total average power is then

$$P_a(f) = \left[X'_r(f) \right]^2 \quad (17)$$

III. EQUIPMENT

A. FLOW SYSTEM

Most of the components of the flow system utilized in this investigation have been used previously as part of a continuing work at the University of Alberta investigating two phase liquid-liquid stratified flow. More specific details concerning the equipment can be found elsewhere⁽³⁾. A line diagram of the flow system itself is presented in Figure 3.

1. Conduit

The rectangular conduit used in this work was constructed of transparent "Plexiglas". Its average aspect ratio as determined by Charles⁽³⁾ from pressure drop measurements, was 7.95:1. The overall length of the channel was 35.7 feet - consisting of a 12.5 foot calming section, a 20 foot measuring section, and a 3.2 foot discharge section. The entry section was constructed so as to cause as little mixing at the interface as possible. The two liquids entered the section at the top and at the bottom of a horizontal circular cylinder, 9 inches long and 9 inches in diameter. Interior baffles prevented the two phases from impinging directly upon each other. The circular cross-section gradually changed to the rectangular cross-section of the channel proper over a distance of 2 feet. This

entry section could be judged to be intermediate between the abrupt and rounded entrances used by Eckert and Irvine⁽¹⁰⁾. The 12.5 foot, or 84 equivalent diameter, calming section was deemed sufficient, based on the work of Eckert and Irvine, to allow the velocity profile to become fully developed. Pressure taps, 1/16 inch diameter openings in the bottom plate along the centre line of the conduit, were situated 12.5 feet and 32.5 feet respectively from the entrance of the channel.

2. Test Section

Built into the conduit was an 18 inch removable test section located 24 feet from the upstream end. This section was previously used for photographic purposes⁽⁴⁾ and for velocity profile measurements^(3 2). The aspect ratio of the test section itself was found to be exactly 8:1 at the point in the cross sectional plane over which the installed pitot tube was free to travel horizontally and vertically. The pitot tube mechanism was utilized as a pointer gauge to measure the position of the interface. This particular piece of the traversing mechanism consisted of a pointer gauge attached to a vertical micrometer screw. A photograph of the traversing mechanism is displayed elsewhere⁽³²⁾.

For photometric work a two-inch diameter hole was cut into the center of the upper plate of the removable test section, 5 inches

from its upstream end. A "Plexiglas" plug, sealed with an O-ring, was positioned into this opening and fastened to the upper plate with three Allen screws. This removable plug allowed easy cleaning of the spot where the light passed through the channel. Thus it was not necessary to drain the channel each time it became necessary to polish the photometric section of the test section.

3. Auxiliary Equipment for the Conduit

The two phases, oil and water, were stored in a combined storage and separator tank. This 14 gauge galvanized steel tank was 52 inches in diameter and 75 inches high, with a capacity of 650 U.S. gallons. Inside the tank were two circular baffles, one being 42 inches in diameter and 52 inches high, and the other being 27 inches in diameter and 26 inches high. These aided the oil-water separation process.

The oil phase, leaving the top of the tank, was circulated throughout the system by a positive displacement gear type pump. This pump, model K30 manufactured by G. D. Roper Corporation, had a capacity of 36 U.S. gallons per minute at 1750 R.P.M. The water phase, leaving the bottom of the tank, was circulated also by a positive displacement type pump. It was a model FL manufactured by Jabsco Pumps Company with a capacity of 30 U.S. gallons per minute at 1750 R.P.M.

Unwanted portions of the total flow were returned to two bypass tanks - one for each phase. These tanks, each having a capacity of 50 U.S. gallons, were installed so that rises in temperature resulting from continued recirculation could be kept at a minimum.

Each of the phases was metered with three rotameters. Upon leaving the rotameters, the fluids passed through a concentric tube heat exchanger for temperature equalization, and subsequently into the channel entry section.

The two phases, upon leaving the conduit, entered tangentially at the midpoint of the separator tank. The oil subsequently overflowed the outer baffle while the water underflowed it.

A sight glass on the side of the tank permitted observation of the levels of the two fluids in the tank. When the oil-water interface dropped such that it was no longer coincident with the vertical centre of the channel, fresh water was added through 0.5 inch openings in the periphery of the tank. Previous work had indicated a problem with calcium deposits in the tank, tubes, and the conduit. An attempt was made to alleviate this problem by installing a model Gn 18 Lindsay Water Softner in the make-up water line. It was placed here because of the high pressure drop across it. At a flow rate of 15 U.S. gallons per minute of water, the pressure drop across the ion exchanger would have been 54 psi, thus resulting in a reduction in the maximum flow rate available.

To remove minute droplets of water which had been entrained with the oil, a 5 inch bed of coarse sodium chloride supported on a perforated tray, had been placed between the outer and inner baffles. In preparing for the experiment it was noticed that severe corrosion in the salt tray itself and in the supporting screen had occurred. The tray was removed, cleaned by sandblasting, and a protective coating applied, which was resistant to salt corrosion. The brass salt retaining screen was replaced with a nylon screen. These modifications eliminated any corrosion and the discoloration of the oil due to the presence of iron ions.

4. Fluids

The two fluids used to conduct this experiment were identical to those used by Charles. The choice of the particular fluids was based on the fact that they had to have a sufficient viscosity differential to cover a wide range of individual flow rates, and that they separate readily under the action of gravity.

The oil, a five to one mixture of Mentor 29 and Bayol 35, was purchased from Imperial Oil Limited. At 80°F, this oil had a viscosity of 4.642 centipoise and a density of 0.8145 gms/cc. The water, at 80°F, had a viscosity of 0.8712 centipoise and a density of 0.9959 gms/cc. The interfacial tension of these two liquids was 42 dynes/cm. The water and oil used in this investigation were slightly

different in that the water had the calcium ions replaced by sodium ions, while the oil had a very low concentration of dye added to it.

Photometric work required that one of the two fluids be dyed to assure the absorption of a certain percentage of light. Previous work⁽²³⁾ disclosed that the dyeing of water with a methylene blue dye resulted in the coagulation of the dye into small particles after a few days. This difficulty was resolved by coloring the oil with Sudan Red 4B A dye. This dye, after it was completely dissolved, resulted in a stable solution having an extremely homogeneous red color which remained unaltered for the duration of the experiment. It was found that 5.2 gms of dye added to 300 U.S. gallons of oil resulted in a solution having a 48.2 per cent transmission of light at 530 m μ as measured on a Spectronic 20 Colorimeter. A more detailed description of the dyeing of the oil and the oil layer thickness versus per cent light transmission calibration is found in Appendix A.

B. PHOTOMETRIC

A schematic diagram of the photometric equipment is found in Figure 4.

1. Light Source

The success of the photometric technique depended on the development of a small diameter, constant intensity, parallel beam of visible light. The per cent transmission of a beam which is an order of magnitude smaller than the length of the smallest disturbance permits the determination of the location of the interface along with a true representation of its interfacial fluctuations at one point.

Figure 4 illustrates the light source and the lens arrangement. A General Electric 1630 six volt bulb mounted in a Bauch and Lomb microscope light source, Catalog Number 33-33-53, supplied the necessary illumination. This source of light was mounted into a 2 inch diameter brass tube which was painted a matte black to eliminate reflections inside the tube. It was found that a minute aperture, combined with a 78 mm focal length lens placed 10.33 inches from the top of the tube, produced a thin beam of light 0.8 mm in diameter. This was approximately one order of magnitude smaller than the smallest regular wavelengths encountered, and insured an accurate representation of the liquid surface structure. A green filter was positioned at the end of the tube to limit the wavelength of

the light beam to approximately $530 \text{ m}\mu$. Previous work⁽⁸⁾ had indicated that this type of filter, combined with the red oil, produced curves of light absorbed versus thickness of fluid that closely adhered to the Beer-Lambert law of light absorption.

A 5-1/4 inch "Lucite" light rod was positioned between the microscope light source and the aperture plate. This "light rod" increased the output light intensity 2-1/4 times, but it was later discovered that rings of light were produced from it. To avoid these changes in light intensity when the light source was moved slightly, this rod was later excluded from the light supply apparatus. The resulting loss in output intensity was compensated for by increasing the power supply voltage to the photomultiplier tube.

The light source itself was powered by a 12 volt wet cell battery. A model 61210 Carter Battery Charger was connected to the battery whenever the light source was in use. This prevented a voltage drain on the battery and insured a constant power supply to the lamp. Small pulsations from the charger were damped sufficiently by the battery, such that the resulting ratio of A.C. to D.C. voltage was approximately 1:1000. The voltage from the battery was reduced from 12 to 6.5 volts by means of two Ohmite Model H rheostats connected in series. These two rheostats, one being 6 ohms, the other 1 ohm, provided excellent control of voltage to the light source.

The complete light source assembly was attached to a specially built frame fastened directly to the wall above the channel. This insured the light source to be secure and free from vibrations or slight movements which could render a change in position with respect to the conduit. The frame was designed so that the whole assembly could be moved vertically or horizontally if necessary. This permitted adjusting the light beam to pass directly through the centre of the photomultiplier tube housing and also the focussing of the beam at the oil-water interface.

2. Light Measuring Device

An RCA Type 6199 Photomultiplier Tube was used to measure the intensity of light transmitted through the channel. This device enabled the light beam to be converted into an electric signal which could be measured with readily available electronic instruments. An expanded circuit diagram of the photomultiplier tube is presented in Figure 6.

Power to operate the tube was supplied by an NJE Corporation model S 325, 500-2500 VDC Power Supply. The photomultiplier anode was maintained at ground potential with the cathode at -850 VDC. Load resistance for the photomultiplier tube output current consisted of a 20K model A Beckman Helipot precision potentiometer. Other details of the tube are available elsewhere⁽³⁰⁾.

The photomultiplier tube was housed in a 12 inch long brass tube having a diameter of 4 inches. The top of the housing consisted of

a piece of "Plexiglas", 0.5 inches thick, with a 1 inch diameter opening in the centre. A concentrated light beam striking the photocathode surface at different positions could lead to serious errors owing to the non-uniformity of the photosensitive surface. To avoid this phenomenon, a frosted glass diffuser was placed into the opening and countersunk in the "Plexiglas" to avoid contact with objects which might scratch, stain, or otherwise render the surface non-uniform. The photomultiplier tube itself was placed 2-1/4 inches below the diffuser, and was wrapped with brass shimstock, which along with the housing, was connected to ground. This shimstock provided electrostatic shielding for the tube which aided in reducing noise generated by stray fields.

3. Electrical Measuring Devices

Figure 4 shows the interconnection of the measuring instruments in a block diagram form.

The time average output voltage across the load resistance was measured by a Non-Linear Systems, Inc. model 481 Digital Voltmeter. It was discovered that even at very low sensitivities, the frequency response of the instrument was too high and fluctuations in voltage readings resulted. To damp out these fluctuations, a 20 μ fd capacitor was placed in parallel with the load resistance and the digital voltmeter for the measurements of the average voltage only. This capacitor damped the fluctuations in voltage sufficiently so that a constant voltage was recorded on the meter. By comparing this output

voltage with the output voltage with the flow shut off, thus with no voltage fluctuations possible, a true average voltage seemed to have been achieved since both readings were within ± 0.5 per cent.

The spectral analysis of the liquid surface disturbances was conducted utilizing a Krohn-Hite Corporation model 330M 0.2 cps-20kcps Band-Pass-Filter and a Flow Corporation model 12 A1 Random Signal Voltmeter. All frequencies below and above the pass-band of interest were filtered out, and the portion of the signal which passed through the filter was measured with the random signal voltmeter, which had a flat frequency response from 2 cps to 250 kcps with a 16 second time constant. The characteristics of this voltmeter were deemed adequate to obtain a true average of the power at the frequencies of the interfacial waves present. A Sola Constant Voltage Transformer, catalogue number 23-22-150, was used to regulate the random signal voltmeter power source in order to supply a constant 118 volts to the meter. This transformer eliminated surges in the supply line, thereby reducing the tendency of the voltmeter to wander excessively.

Preliminary investigations with a Tektronix type 502 Oscilloscope indicated that the noise level was quite high relative to the output signal. An attempt to remedy this undesirable situation was made by replacing the leads with shielded leads, all the mesh shields of which were grounded to the NJE Power Supply to eliminate the possibility of ground loops. These leads were cut to the proper length, to fit between the

instruments. All connections were cleaned and soldered carefully, and all instruments were plugged in the same grounded electrical outlet. This little extra care and precision reduced the noise level by approximately a factor of ten.

C. AUXILIARY MEASURING DEVICES

The temperatures of the fluids were measured with four thermometers present in the flow system. Two mercury thermometers were placed into openings in the side of the storage tank - one in the oil and one in the water, while two dial thermometers were present in the entrance section of the channel.

Flow rates were monitored with a set of six rotameters - three for each phase. The two larger ones for each phase were Brooks Rotameters while the smaller one for each phase was a Fischer-Porter Rotameter.

Pressure drop measurements across the 20 foot test section were taken with a manometer constructed of 10 mm glass tubing. The leads from the channel consisted of plastic tubing. This simple manometer was quite adequate provided a suitable manometer fluid was chosen. The manometric fluid used was a 2.5:1 mixture of benzene and carbon tetrachloride. These fluids had approximately the same vapor pressure which guaranteed a constant density, even with evaporation which occurred during handling. Their solubility in water has been reported as 0.08 parts per 100 at 20°C. This solubility was considered low enough so that changes in density in the water layer above this organic liquid mixture could be neglected. The rapid evaporation of the liquid to the atmosphere made density measurements extremely

difficult. The density was finally determined on a L. W. Hohwald Chain Gravitometer, checked with an ordinary hydrometer, and found to be 1.126 gms per cc at 78°F. This mixture was dyed slightly red for convenience of observation.

The height of fluid in the manometer tubes was measured with a cathetometer. With the cathetometer it was possible to read a level difference to an accuracy of ± 0.005 cm.

IV. PROCEDURE

A. FLOW SYSTEM

1. Preliminary

Prior to conducting this investigation, the storage tank along with the channel was cleaned thoroughly. The tank was then filled with water and the water circulated throughout the system to rinse out the piping. This water was drained and fresh water added. The oil was added and subsequently dyed. The oil surface in the separator was positioned approximately four inches above the oil outlet while the oil-water interface was positioned at a point corresponding to the mid-point of the conduit. This provided a sufficient head above the oil outlet to prevent the formation of the small air bubbles in the oil phase, which were caused by the oil pump drawing air.

Before adding the dye to the oil it was dissolved in two gallon quantities of oil and these concentrated solutions poured into the tank. This prevented the dye, which took about one day to dissolve, from falling right through to the oil-water interface. A better control of the dye concentration could be obtained in this manner.

The rotameter calibrations were checked and found to agree with those determined by Charles within ± 0.8 per cent. Rotameter readings are to be taken using the top of the float of the three oil rotameters and small water rotameter, and the middle line present on the floats of the two larger water rotameters.

2. Experimental

Prior to starting the flow through the conduit, the inlet water and oil valves along with the vent on top of the conduit were opened, and the two phases were allowed to fill the channel under gravity. Care was taken not to allow the oil phase to come in contact with the bottom of the conduit since the "Plexiglas" is preferentially oil wetting. This also prevented the oil phase from entering the manometer lines. After the oil had reached the top of the conduit, the water and oil pumps were started. The rotameters were first set so that a high flow rate prevailed. This assured that the entire channel was completely wet with either one of the fluids. After the channel was uniformly wet, the flow was set by adjusting the by-pass valves on the respective pumps along with the relative rotameter valves. Daily, before turning it on, the water pump was greased with Andok BR FMCO Hi Temp Water Pump Grease. The oil pump did not require lubrication.

B. ELECTRONIC SYSTEM

1. Preliminary

Most of the preliminary work with the electronic system is described in Section III. The preliminary work consisted basically of connecting the instruments as illustrated in Figure 4, getting them operational, and subsequently attempting to reduce the noise level as described in Section III-B-2.

2. Experimental

To obtain a basis on which to calculate the height of oil in the channel, the oil and water flow rates were set such that laminar flow in both phases prevailed. This assured a smooth interface between the two liquids. The light source, along with the battery charger, was turned on. The light source was given one hour to warm up in order to assure a constant intensity light supply during the actual experiment. Concurrently with the light source, the random signal voltmeter was switched on. It too was given one hour to warm up so that it might reach thermal equilibrium. The band-pass-filter and the digital voltmeter were also made operational at this time.

After allowing about one half hour for the flow to reach steady state, height measurements were taken with the pointer gauge traversing mechanism described earlier. Twenty such measurements were taken and the results averaged. Immediately upon completing the height

measurements, the lights in the corridor were extinguished, and the power supply to the photomultiplier turned on. After approximately fifteen minutes, which was the time for the output signal, as measured with the digital voltmeter, to become constant, the digital voltmeter reading was recorded. Using this voltage reading plus the height measurement, the unknown V_0 in Equation 36 could be determined for subsequent turbulent runs.

As quickly as possible after taking the voltage reading, the oil and water flow rates were set at the desired levels. The digital voltmeter reading was taken over a short interval, and this reading was used to calculate the new average height of the oil at that flow rate.

The random signal voltmeter was then zeroed and calibrated. The output signal from the photomultiplier tube was switched to pass through the band-pass-filter and on to the random signal voltmeter. It was discovered that excessive wandering of the random signal voltmeter was diminished if the upper and lower cutoff frequencies on the band-pass-filter were set at the same value. The initial reading was taken with the upper and lower cutoff frequencies set at 54 cps to eliminate any possibility of 60 cps pickup. Subsequent RMS voltage readings were taken with the random signal voltmeter as the setting on the band-pass-filter was decreased. Approximately two or three minutes were allotted for each reading to allow the random signal

voltmeter time to stabilize. This prevented the transients caused by changing the setting on the band-pass-filter from throwing the random signal voltmeter out of calibration. To assure that no gross changes had taken place in the system, the digital voltmeter reading was checked periodically. If a variation in the signal strength had occurred, the flow rate was checked. If it had not changed, the proper signal strength was obtained by increasing or decreasing the Helipot load resistance as required. This procedure was followed down to the band-pass-filter setting of 2 cps which was the lower limit of the linear operating range for the random signal voltmeter. Frequencies lower than this caused serious fluctuations in the indicating needle, and thus no reliable readings at lower frequencies could be obtained. At the end of the series of spectral measurements the upper and lower cutoffs of the band-pass-filter were set at 2 cps and 54 cps respectively to obtain the total power present in this frequency band.

The power supply to the photomultiplier tube was switched off and the lights in the room turned on to permit the pressure drop measurements for this combination of flow rates. The lights were again turned off, and the power supply turned on. A few minutes were allowed for the photomultiplier tube again to warm up such that the same reading as previously obtained on the digital voltmeter was realized. When this condition was reached, the flow rates of oil and water were adjusted to the next desired value; the reading on the digital voltmeter noted; and an identical measuring procedure followed.

C. SHUT-DOWN

The shut-down procedure was commenced by turning off the power supply, turning on the lights, and in turn shutting off all of the electrical apparatus. The two pumps were shut off together with the channel exit valve. The two inlet valves were then shut as quickly as possible. Periodically a height reading was taken at this point as sort of a "stop flow" check on the measurements obtained with the digital voltmeter. The channel was then drained slightly to relieve any pressure strains, with care taken not to allow the oil to come in contact with the channel bottom.

V. TREATMENT OF DATA

A. FORMULATION OF THE AVERAGE POWER SPECTRUM

As mentioned earlier, the required electrical signal was generated by passing light through the liquids and converting it to an electrical current by means of a photomultiplier tube. Since the photomultiplier tube was operated at the same set of conditions throughout the entire experiment, the output voltage could be assumed to be directly proportional to the intensity of the light impinging onto the photocathode. That is:

$$V = CI \quad (18)$$

The problem now is one of relating this voltage across the load resistance to the surface elevation in the channel proper. From previous calibration runs⁽⁸⁾, oil colored with Sudan Red 4B A dye was found to obey the Beer-Lambert law of light absorption which relates the fraction of light transmitted to the thickness and color concentration of the fluid in question⁽³⁶⁾. This fact was verified during the present investigation. A complete description of the preliminary calibrations is found in Appendix A.

The Beer-Lambert law states that

$$I = I_0 e^{-ach} \quad (19)$$

with a being an absorption coefficient of unit concentration and c

being the concentration. Preliminary investigations indicated the possibility of combining these two coefficients into another coefficient k . Combining Equations (18) and (19) results in a relationship between the output voltage and the thickness of the fluid h - in this case oil - in the path of the light.

$$\frac{V}{V_o} = \frac{I}{I_o} = e^{-kh} \quad (20)$$

The dye concentration varied from run to run due to slight discoloration of the oil after prolonged circulation along with periodic addition of fresh oil. For this reason k had to be found as a function of some known parameter so that the output voltage and the height of the oil could be related for any dye concentration. The parameter chosen was per cent transmission of light at 530 $m\mu$. This resulted in the crossplot of per cent transmission versus k illustrated in Figure 8.

Utilizing a value of k from Figure 8, dictated by the per cent transmission for that particular run, the height for any voltage output could be determined as long as one value of height along with its corresponding voltage was known. This one value of height was found by means of a needle placed into the fluid. For this known height, the voltage output was taken which enabled the value of V_o to be calculated from Equation (20). Thus, after a basis V_o had been

determined, the height of the oil, whether it was the root-mean-square perturbation height or the average height, could be calculated knowing its corresponding voltage.

The difference h' between the instantaneous and the average film thickness is given by

$$\begin{aligned} h'(t) &= h(t) - \bar{h} = -\frac{1}{k} \left[\ln \frac{\bar{V} + V'(t)}{V_0} - \ln \frac{\bar{V}}{V_0} \right] \\ &= -\frac{1}{k} \left[\ln \left(1 + \frac{V'(t)}{\bar{V}} \right) \right] \end{aligned} \quad (21)$$

If $\ln \left(1 + \frac{V'(t)}{\bar{V}} \right)$ is expanded in a Maclaurin's series, it becomes

$$\ln \left(1 + \frac{V'(t)}{\bar{V}} \right) = \frac{V'(t)}{\bar{V}} + \frac{1}{2} \left(\frac{V'(t)}{\bar{V}} \right)^2 + \frac{1}{6} \left(\frac{V'(t)}{\bar{V}} \right)^3 + \dots \quad (22)$$

If it can be assumed that $V'(t)$ is very much less than $\bar{V}$, then it could be said that

$$h'(t) = -\frac{1}{k} \frac{V'(t)}{\bar{V}} \quad (23)$$

As it turns out in the worst possible case, $\bar{V}'$ is 1.5 per cent of $\bar{V}$. This results in a 0.0011 per cent error. Therefore, Equation (23) was assumed to apply in all cases.

The concept of power spectral analysis can be used to help characterize the waves formed on the interface between two liquids in co-current flow. For this application, the perturbation signal $X'(f)$ introduced in the theory is replaced by the perturbation surface

elevation $h'(f)$. This results (from Equation (17)), in the average power at any frequency being defined as

$$P_a(f) = \left[h_r'(f) \right]^2 \quad (24)$$

If Equation (23) is substituted into Equation (14), with X being replaced by h , the root-mean-square perturbation height becomes:

$$h_r' = \frac{1}{k} \frac{V_r'}{\overline{V}} \quad (25)$$

while the average power spectral density becomes, from Equation (24)

$$P_a(f) = \left[\frac{1}{k} \frac{V_r'(f)}{\overline{V}} \right]^2 \quad (26)$$

The time average oil height $\bar{h}$, defined as

$$\bar{h} = \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T h(t) dt \quad (27)$$

was measured with a digital voltmeter, as described earlier.

Other statistical functions, based on the spectral density $P(f)$ may be used to characterize further the interface, if the system can be assumed to be a stationary Gaussian process. It was shown by Lilleleht⁽²³⁾ that for air-water flow, the system was extremely close to being Gaussian, particularly at lower frequencies. One function which is of interest is the number of zero crossings $N(\bar{h})$, or the number of times the surface crosses the mean level $\bar{h}$ per second.

$$N(\bar{h}) = 2 \left[\frac{\int_0^\infty f^2 P(f) df}{\int_0^\infty P(f) df} \right]^{1/2} \quad (28)$$

Another useful function is the number of maxima or minima $N(\bar{h})$, which is the number of wave crests and wave troughs respectively, that can be expected to pass a fixed point per second.

$$N(h) = \left[\frac{\int_0^{\infty} f^4 P(f) df}{\int_0^{\infty} f^2 P(f) df} \right]^{1/2} \quad (29)$$

These were calculated by integrating, using Simpson's rule for numerical integration, the true power spectral density function which was obtained.

B. TREATMENT OF NOISE

As described in the section dealing with experimental procedure, noise posed a problem with regard to the determination of the power spectrum. In order to obtain a truer representation of the power spectrum, the electrical noise generated in the photomultiplier tube, the band-pass filter, and the random signal voltmeter itself should be subtracted from the raw voltage reading.

The root-mean-square noise level was determined over the range of frequencies with no flow in the channel. Thus the noise was observed with no contributions of voltage from fluctuations in surface elevation. Because the noise level increases with an increase in output voltage, and since h_r' is inversely proportional to the average output voltage, it was deemed best to convert the noise contribution into its equivalent value of root-mean-square perturbation height before subtracting it from the raw height.

The equation for the true root-mean-square height then becomes

$$h_r' = \left[\frac{1}{k} \frac{V_r'}{\bar{V}} - \frac{1}{k} \frac{(VN)_r'}{(\overline{VN})} \right] \quad (30)$$

(VN) being the contribution of the noise to the total voltage.

C. TREATMENT OF THE POWER SPECTRAL DENSITY EQUATION

After subtracting the noise contribution, the resulting perturbation height was squared. This resulted in the average power spectral data - $P_a(f_M)$ in Equation (11), which is also given below. This equation must be solved for the true power spectral density $P(f)$.

$$P_a(f_M) = \int_0^{\infty} 2 \left| Y(f, f_M) \right|^2 P(f) df \quad (11)$$

The value of the transfer function $Y(f, f_M)$ was given by the manufacturer of the band-pass-filter as

$$Y(f, f_M) = \frac{\left(\frac{\sqrt{n} f}{f_M} \right)^4}{\left[1 + 2iA \left(\frac{\sqrt{n} f}{f_M} \right) - \left(\frac{\sqrt{n} f}{f_M} \right)^2 \right]^2 \left[1 + 2iB \left(\frac{f}{\sqrt{n} f_M} \right) - \left(\frac{f}{\sqrt{n} f_M} \right)^2 \right]^2} \quad (31)$$

where n is f_2/f_1 ; f_M is $\sqrt{f_1 f_2}$, f_1 and f_2 are the lower and upper cut-off frequencies respectively, and A is a peaking factor given by the manufacture as 0.6.

Knowing this transfer function and also the average power in the pass-band $P_a(f_M)$, an integral equation of the first kind must be solved for the power spectral density $P(f)$.

If the upper and lower cut-off frequencies are made equal, that is,

$$f_1 = f_2 = f_M \quad (32)$$

and letting

$$H(f, f_M) = 2 \left| Y(f, f_M) \right|^2 \quad (12)$$

Equation (11) becomes

$$P_a(f_M) = \int_0^{\infty} H(f, f_M) P(f) df \quad (33)$$

At any particular filter setting f_M , $H(f, f_M) P(f)$ must be integrated over all frequencies f to obtain the average power at f_M . This integral converges quite rapidly due to the nature of the transfer function. An illustration of the behavior of the transfer function plotted against f/f_M with n as a parameter, is plotted in Figure 22. The averaging time for $P_a(f_M)$, dictated by the time constant of the random signal voltmeter, was deemed sufficient to give a good average value of the power output.

A first approximation to the solution of this equation has been proposed by Wonnacott⁽³⁵⁾ and used by others^(7, 23) in proposing the power spectral density of waves. This approximation involves the assumption of a constant spectrum over the frequencies included in the range of the transfer function and results in the following equation.

$$P_a(f_M) = P(f_M) \int_0^{\infty} H(f, f_M) df \quad (34)$$

This form of solution, albeit being sufficiently accurate over part of the spectrum, is in error at higher frequencies f_M where the

power transfer function covers a wider range of frequencies, and at such points the whole spectral density curve is involved in the computation. For example, at $f_M = 1$ cps, the power transfer function is significantly different from zero over the range $0.2 < f_M < 2.2$ cps while at $f_M = 20$ cps, the transfer function is significant over the range $4 < f_M < 44$ cps. This method was tried and the results for $P(f)$ gave a fairly accurate representation of the total power present. The values for $P(f)$ though, when substituted back into Equation (33) and integrated gave values not quite as close to the desired values of $P_a(f_M)$ as other techniques employed to solve this equation. This is illustrated in Table 17. The method in which this back substitution was done is described in a later section.

In order to obtain a more accurate representation of the spectral density, it was decided not to consider the density as constant over the frequencies encompassed by the transfer function but instead to solve Equation (33) as closely as possible to the true analytic approach with a minimum of basic assumptions.

Several ideas and methods which seemed attractive at first were later found to be not proper at all. Two of these methods which reached preliminary levels of calculation will be discussed briefly so as to prevent these same pitfalls by later workers. One procedure attempted was first to fit the complicated transfer function

with a ninth order orthogonal polynomial using Forsythe's method⁽¹¹⁾. Over a small range of f it was decided to approximate $P(x)$, not as a constant, but as a third order polynomial. The resulting twelfth order polynomial formed when the two polynomials were multiplied was integrated analytically. The limits were substituted and the result was a fourth order polynomial in f_M with four unknown parameters. Four equations were evolved for each section which were solved by Gaussian elimination⁽²¹⁾ for each segment. This method did not work for the same reason that the assumption of constant spectral density does not apply; this reason being that the curve is being integrated over all frequencies while the spectral density is being approximated at only one point for the constant density approximation, or as in the latter assumption, over a small range. This latter method is extremely bad in that the approximation for the spectral density really gets out of hand when the original approximating range is exceeded. This is probably the reason the first approximation gives a fairly good value for total power, that is, the low values assumed may cancel the high values assumed where in the latter case they may never cancel depending on the shape of the approximating polynomial.

The next logical step is to fit the spectral density curve completely with some higher order polynomial, multiply it by the

transfer function, and then solve for the unknown parameters with a least squares criterion. This was tried but no polynomial of any degree up to nine could be found to fit the spectral curve with any accuracy at all. Above this degree the matrices evolved from the least squares criterion became difficult to solve due to ill condition effects. This method also did not result in a convergent solution. Perhaps if some statistical function such as a normal distribution for the transfer function, and a chi squared or gamma distribution for the spectral density, was used to fit the curves, a better solution might be evolved with this technique.

D. SOLUTION OF THE POWER SPECTRAL DENSITY EQUATION

To evolve a solution from Equation (33), an iterative procedure was utilized in an attempt to force fit the power spectral density to the experimentally determined average power spectral data. The problem with this equation is that of solving implicitly for the power spectral density which is within the integral, rather than solving explicitly for the left hand side. Since an analytical solution of the equation is impossible, it was judged best to solve it utilizing the IBM 7040 computer available at the University of Alberta Computing Centre.

The calculation procedure was initiated by placing a smooth curve through the experimental data. This assured a continuous curve to which the right hand side of Equation (33) could converge. These data, along with the noise data, were read in at each whole number value of frequency from 1 cps to 100 cps. The last 45 pieces of data were dummy values required in the calculational procedure.

The calculational procedure involved the integration of the product of the transfer function and the unknown power spectral density. In this procedure a function was initially assumed for the power spectral density $P(f)$. For each value of f_M the range of frequencies traversed by the transfer function were divided into 10 equal intervals, four to the left of f_M and six to the right. This

covered the range of values of the transfer function for $H(f, f_M) < .005$, with $.2 < \left(\frac{f}{f_M}\right) < 2.2$. Values of $\left(\frac{f}{f_M}\right) > 2.2$ yielded insignificant values of $H(f, f_M)$. Both the assumed spectral density curve and the known transfer function curve were approximated by first order curves over the small interval. The unknown parameters were solved for and these two curves were subsequently multiplied together yielding a second order curve which was integrated analytically, and the limits of the interval under consideration substituted into the resulting equation. This method was followed over the ten intervals, and the separate integrations summed to give the calculated value of $P_a(f_M)$. This procedure was repeated for each whole numbered frequency from 1 cps to 45 cps. This resulted in 45 calculated values of $P_a(f_M)$. The calculated and experimental values of $P_a(f_M)$ were compared and a correction applied to the assumed $P(f)$ to decrease or increase it as the case might be in order that a better calculated value of $P_a(f_M)$ might be obtained. The new $P(f)$ was found at each of the 45 points by the following equation:

$$P_{\text{new}}(f) = \frac{P_a(f_M)}{P_c(f_M)} P_{\text{old}}(f) \quad (35)$$

The values of $P_{\text{new}}(f)$ at frequencies above these 45 values were determined using the value of $P_a(f_M) / P_c(f_M)$ found at 45 cps. This method of correction resulted in a smooth curve for the newly assumed

$P(f)$ and also lowered the values of $P(f)$ if $P_c(f_M)$ was high and vice versa if $P_c(f_M)$ was low. This complete procedure was iterated ten times after which time the change in the assumed $P(f)$ was very slight. After the tenth iteration, the total power under the spectral density curve $P(f)$ was determined by integrating the curve using Simpson's rule. The result was multiplied by two to give the true calculated power, according to Equation (8), and compared with the total power measured experimentally. The results are illustrated in Tables 2 - 16.

To test the effect of a varying pass-band width, two values of n were applied to the transfer function for the first run. The total power obtained and the calculated values of the spectral density were compared with those obtained for $n = 1$. The modifications required to accomplish this, along with a more complete description of the computer program are described in Appendix B.

VI. RESULTS AND DISCUSSION

A. FLOW REGIME TRANSITIONS

The flow regimes concomitant with stratified flow, along with their corresponding interfacial structures observed in this investigation, were similar to those observed previously⁽³⁾, and illustrated in Figure 1. Albeit, the exact flow conditions for the inception of these regimes was not the main objective of this work, the observations which were made indicated agreement with the earlier results.

The inception of the two-dimensional waves is difficult to detect precisely, even with a light absorption technique, due to the presence of some turbulent patches in the water layer at Reynolds' numbers as low as 1200. It was discovered that these turbulent patches, evident because of the disturbed interface coincident with their inception, were preceded by an increase in thickness of the water layer of 0.010 - .015 inches. As their intermittency factor increased to unity, a continuous wavy surface evolved. Thus, even with light absorption measurements, the exact conditions under which the surface became completely wavy still depended on the judgment of the observer, since these surges upset the steady state condition of the random signal voltmeter, which due to its high time constant, required a long period to again reach equilibrium. Just as good an

approximation of two-dimensional wave inception could probably be obtained by visual observations.

The roll waves, described previously, were impossible to pick out. The flow regimes in which these waves were proposed to exist appeared to give the same power spectra as the large two-dimensional waves. Under the flow rates attainable with the present equipment, stratified flow patterns could probably be described as well with four wave regimes, omitting the roll wave regime altogether.

B. AVERAGE HEIGHT MEASUREMENTS

A plot of the average oil film thickness $\bar{h}$ versus the superficial water Reynolds' number with the oil Reynolds' number as a parameter is illustrated in Figure 9. The dashed lines show the oil film thickness as measured by Charles⁽³⁾. The slopes tend to increase as the oil flow rate is decreased indicating a stronger effect of the water flow rate on the oil thickness at lower oil flow rates. The plot for the turbulent-turbulent case, though, curves sharply downward at high values of the Reynolds' number of the water phase. The last two points on this curve should probably be discounted because of the problem of emulsion formation which occurred at the high oil flow rates and which was unnoticed until after the data at these points had been taken. To avoid this emulsion problem, the three-dimensional wave regime was not investigated further because of the time required for the emulsion to break. All other points, though, should be correct as height measurements obtained with the same flow rates but on different occasions agreed within 1 - 2 per cent, which corresponds to 5/1000 of an inch.

A comparison of the height values with those obtained by Charles reveals that the present height measurements are higher in the laminar-laminar regime and lower in the laminar-turbulent regime. This could be explained by possible inaccuracies in the

"stop flow" type of measurement utilized by Charles. When "stop flow" measurements were taken during the present investigation, they too, were higher than those obtained by photometric means. It was discovered also that as time progressed after the flow was stopped, the oil height increased. This was due to the oil in the entry section tending to seek its own level. Depending on the time interval between stopping the flow and obtaining a height reading, the resulting measurement was different. This, coupled with a technique utilized by Charles to obtain the height measurements which could be influenced by meniscus effects, would tend to account for the resulting discrepancies in the two sets of data.

As mentioned in the previous section, the transition to turbulence in the water phase corresponded to an increase in the thickness of the water layer. This increase in insitu ratio - defined as the ratio of the height of water to the height of oil present in the conduit - was prevalent throughout the laminar-turbulent region. This increase in insitu ratio is illustrated in Figure 10 which is a plot of insitu ratio versus input ratio. In the small two-dimensional wave regime the points appeared slightly above the theoretical laminar-laminar curve⁽³⁾. As large two-dimensional waves appeared, the insitu ratio showed an increase above the laminar-laminar curve which increased further as the water flow rate was increased. This is

due to the high rate of energy dissipation in the turbulent water layer causing the water height to be relatively higher than a corresponding laminar case. It can also be observed in Table 1 that at turbulent oil flow rates the insitu ratio is less than the input ratio. This effect is tantamount to the turbulent water case, except that turbulence in the oil layer causes a higher rate of energy dissipation because of its higher viscous forces.

It may be noted that the insitu ratios obtained for the laminar-laminar case fall directly on the theoretically predicted laminar-laminar curve. This would illustrate that the measurements taken are probably quite accurate, especially for the laminar-laminar case.

C. POWER SPECTRAL DENSITY MEASUREMENTS

The flow conditions at which power spectral density curves were obtained are illustrated in Figure 1. An attempt was made to cover the entire flow map available within the capacity of the equipment. The frequencies at which power spectral readings were obtained ranged from 2 cps to 54 cps. The raw voltage output data is presented in Tables 2 - 16. Prior to reading these data into the computer program, described in Appendix B, the data were plotted and smoothed in order to reduce the probability of the iterative procedure from running out of control.

It was discovered that the random signal voltmeter was much more stable when the upper and lower cut-off frequencies of the band-pass filter were set at the same values. To test the effect of a varying pass-band width, power spectral curves were obtained for Run 1 with the ratio f_2/f_1 set at 1.0, 1.2 and 1.5. The effect of a wider pass-band on the average output voltages is illustrated in Figure 12. Another example of a typical curve of the output voltage as a function of the filter pass-band frequency is shown in Figure 13.

According to Equation (8), it is possible to calculate the total power in the frequency range. This provides a check for the accuracy of the calculated total power against that measured over the complete frequency range investigated.

The corrected final curves for the power spectral density function $P(f)$ obtained from the computer are shown in Figures 14 - 19, and the data tabulated in Tables 2 - 16. Tables 2 - 16 also show the values of $P_a(f_M)$ obtained from Equation (37). Substituting the values of $P(f)$ back into Equation (33) results in a calculated value of the average spectral density $P_c(f_M)$. If $P(f)$ is the proper value of the spectral density, $P_c(f_M)$ should equal $P_a(f_M)$. This provides another check on the validity of the calculated spectral density.

The maximum frequency in the power spectral density curve gives the frequency of the components containing the most potential energy. The spectral densities in the small two-dimensional wave regime each have one maximum and a much lower root-mean-square wave height than the large two-dimensional waves. This is concomitant with a low total power observed with the small waves. In the three-dimensional wave regime the spectral density curves were quite flat but contained a relatively high total power indicating an almost uniform distribution of frequencies illustrated by Curve 5 in Figure 15. The high total power present shows the waves to be of a high amplitude in this regime. Their relative magnitude is illustrated in Table 1. The large two-dimensional regime is interesting in that two and, at high water flow rates, three maxima were observed. The third peak could be due to the calculation procedure, but, as demonstrated in

Table 16, at frequencies between 20 cps and 26 cps, the raw data tended to show signs of existence of another peak. This was smoothed out when the data were smoothed, but the third peak reappeared in the true power spectral density function. This plurality of dominant frequencies shows that, in fact, smaller high frequency waves actually are superimposed upon larger ones of a lower frequency as has been observed visually previously⁽³⁾. It is also interesting to note that the spectra seem to approach the f^{-5} law as suggested theoretically by Phillips⁽²⁸⁾, as an equilibrium condition is approached.

In the two-dimensional wave regime, at least one of the peak frequencies increases monotonically with the mean water velocity at each particular oil Reynolds' number. This is demonstrated in Figure 21. Each curve tends to have the same slope indicating that the rate of increase of dominant frequency is independent of the oil flow rate. This could also indicate that the small two-dimensional waves are caused by turbulent fluctuations in the water layer, since the frequency at which waves first appear lies on this line, and this frequency increases with mean water velocity, indicating that it is directly proportional to the mean water flow rate.

The rate of movement downstream of turbulent fluctuations is also proportional to the mean water flow rate. This figure also illustrates values of dominant frequency which lie very much below

the small wave line. These frequencies are those of the large two-dimensional waves which would appear to be travelling slower than the actual water flow and seem to be caused by some instability in the mean flow.

As mentioned earlier in this section, the fact that the power spectral density can be integrated to obtain the total power, which in turn can be compared with the measured total power, provides a convenient check on the accuracy of the calculated power spectral density. This was done using Simpson's rule for numerical integration. A comparison of results obtained utilizing various techniques for obtaining the power spectral density function are illustrated in Table 18. The method finally chosen for correcting the crude spectral data, yielded calculated values of total power which fluctuated ± 3 to 5 per cent about the measured value. When the spectral density was calculated at every fifth value of frequency, the results were not as good as those with calculations at every measurement frequency, because of the errors which arose in the critical region between 1 cps and 5 cps. The total power using the method of assuming a constant spectral density, Equation (34), results in values of total power as good as those obtained by the iterative computer solution. This method of approximation was discarded, however, because the values of $P_c(f_M)$ obtained in this manner were not as close to $P_a(f_M)$

as those obtained for $P_c(f_M)$ by the iterative technique were, for any of the curves calculated. This is due to the effect of change in slope of the power spectral density curve within the range of frequencies encompassed by the transfer function. This is particularly true at higher frequencies f_M where a wider range of frequencies is effectively covered by the transfer function. This is shown in Table 17.

The total power calculated using a pass-band width greater than 1.0 resulted in the total power calculated deviating further from the measured value than in the case for both upper and lower cut-off frequencies set at the same value. In fact, the deviation for an n of 1.2 and an n of 1.5 was 70.9 per cent and 60.4 per cent respectively, as shown in Table 18.

The root-mean-square surface fluctuations h_r' about the mean oil film thickness $\bar{h}$ were calculated and plotted in Figure 20. Although h_r' consistently increased with water Reynolds' number, no definite trend in h_r' could be noticed with the data available. This is due to the wide range of flow rates investigated in this general attempt of covering the entire flow map. In the large two-dimensional wave regime, h_r' appeared to increase as the oil flow rate decreased, while in the small two-dimensional wave regime, the opposite was true in that h_r' increased along with the oil flow rate. This is probably caused by the fact that at lower superficial oil Reynolds'

numbers, the large two-dimensional wave regime is entered at lower superficial water Reynolds' numbers. This being the case, the waves resulted in having a better chance of growing at lower superficial oil Reynolds' numbers. This effect is shown in Figure 1.

The number of times the interface crosses the mean interfacial level $N(\bar{h})$ and the number of maxima or minima passing a point per second $N(h_M)$ were calculated for each run using Equations (28) and (29), which have been derived for a Gaussian surface. The results of this calculation are presented in Table 19. From the limited data available, no definite conclusions can be drawn regarding the behavior of $N(\bar{h})$. The values of $N(h_M)$, though, appear to increase monotonically with the mean velocity of the water phase in a manner similar to that of the increase in the dominant frequency with the water flow rate. This is illustrated in Figure 21 where $N(h_M)$, which should be proportional to the number of waves passing a point per second, is plotted against the mean water flow rate. It is interesting to note the $N(h_M)$ increases at a greater rate than that of the dominant frequency. The line, too, falls above the lines for the dominant frequencies assumed to be caused by turbulent flow and those assumed to be caused by instability in the mean flow. The value for $N(h_M)$ appears, then, to be summation of the two or three dominant frequencies present. This is to be expected since both values were derived from

the same power spectral density curve. The greater rate of increase of this curve may indicate the formation of more waves as the water flow rate increases. Not enough data, however, were available to judge whether the system under consideration was Gaussian or not.

In summing up, two basically different mechanisms seem to be operative in generating waves at liquid-liquid interfaces. In attempting to characterize the interfacial structure by power spectral analysis, it was noticed that at low oil flow rates, and with the water barely in the turbulent region, extremely minute fluctuations appeared at the interface. These minute fluctuations contained no experimentally measurable power, that is, no maximum could be perceived. The power spectra for these disturbances was just a straight line, that is, the power observed at each frequency was the same and relatively small in absolute value. This would indicate that these disturbances have no dominant frequency but are a random combination of all frequencies. It was also noticed visually that these disturbances travelled downstream with a velocity identical to that of the interface. As the water velocity was increased, a perceptible maximum became evident in the power spectrum. As the flow rate of water increased, this maximum became greater in both magnitude and frequency, indicating the influence of the increases in the degree of turbulence and the convection velocity of turbulent fluctuations, respectively.

Further to this conclusion, in the region of large two-dimensional waves, two maxima appeared in the power spectrum. The maximum for the original disturbances had shifted to a higher frequency while a new maximum appeared at a lower frequency. This would indicate that the large waves were travelling at a lower velocity than the interface. These curves are shown in Figures 14 - 19.

Thus, Phillips' resonance mechanism seems to hold for the initial disturbances while some instability of mean flow theory may be responsible for the appearance of the larger waves.

As illustrated in Figure 21, the dominant frequency plotted against mean water velocity yields a straight line. This indicates that the dominant frequency of the waves caused by turbulent fluctuations is proportional to the average velocity of the water. This concept, too, is illustrated by Charles and Lilleleht⁽⁵⁾, where a graph of wave velocity versus average water velocity yielded a straight line with a slope of one for the initial small waves.

VII. RECOMMENDATIONS FOR FURTHER WORK

The computer program developed to solve for the power spectral density gave results which appeared to be quite good, based upon total power measurements and the comparison of true and calculated values of the power spectral density at each frequency. Other methods might be employed, though, in attempting to eliminate discontinuities which are liable to arise with the iterative procedure used here. Solodovnikov⁽³³⁾ mentions several methods of approximating the spectral density function with a meromorphic function; or it might be approximated with some statistical function. The chosen approximating function could be multiplied by the transfer function, integrated, and the unknown parameters obtained by either a least squares or a linear programming technique. The calculational procedure could get quite involved, though.

The spectral density function could be compared with its corresponding autocovariance function obtained independently. This is also explained in Solodovnikov⁽³³⁾ and would provide a means of checking the measured power spectrum.

One measurement which would be extremely valuable in characterizing the interfacial waves would be the velocity distribution at the flow rate under investigation. This would enable the determination of the friction velocity and as such, allow the dimensionless correlating

parameters developed by Lilleleht⁽²³⁾ to be utilized. The interfacial velocity would also be valuable in an attempt to show the relationship between the turbulent fluctuations and the wave frequency. It is not recommended that the interfacial velocity be determined by the injection of dust on the interface, as this would lead to real problems of accuracy with the photometric technique.

Along the same line of proving the cause of inception of waves to be that of turbulent fluctuations, one extremely useful piece of information would be that of either the wave velocity or the wavelength. This would enable the relationship between the velocity of the turbulent fluctuations and of the waves to be resolved. With this information, the power spectra which have been obtained for turbulent fluctuations could be compared with those obtained for surface fluctuations. The wavelength of the waves might be determined through the installation of a second light channel. A variance of distance between these could result in measurements relating the wavelength to the flow parameters.

Prior to making any detailed conclusions regarding the interfacial characterization of oil and water, further measurements should be conducted, concentrating on specific regimes. This investigation was an attempt at finding out generally what is happening throughout the complete flow map. More detailed investigations of the large two-dimensional wave regime would prove interesting. A better correlation

of the plurality of peaks could be obtained with more runs taken at oil Reynolds' numbers between 100 and 400 and water Reynolds' numbers between 10,000 and 22,000. These combinations of flow rates should be possible before the oil layer becomes too thin to allow accurate measurements.

The case of turbulent oil and turbulent water should be avoided with the present system, due to the problem of oil emulsification at high oil flow rates.

VIII. NOMENCLATURE

a	absorption coefficient of unit concentration in Beer-Lambert law
A	peaking factor in transfer function; given by the manufacturer to be 0.6
c	concentration term in Beer-Lambert law
C	autocovariance function
f	frequency (cps)
f_1	lower cut-off frequency on band-pass filter (cps)
f_2	upper cut-off frequency on band-pass filter (cps)
f_M	midband frequency defined as $\sqrt{f_1 f_2}$ (cps)
$\bar{h}$	time average oil layer thickness (in)
h'	difference between the oil film height and the time average oil film height (in)
h_r'	root-mean-square of this perturbation
(hN)	height equivalent of noise output (in)
H	power transfer function = $2 \left Y(f, f_M) \right ^2$
I	intensity of impinging light on photomultiplier
I_0	intensity of light with nothing in the path
k	dye concentration sensitivity
n	pass-band width defined as f_2 / f_1
$N(\bar{h})$	number of crossings per unit time at $\bar{h}$
$N(h_M)$	number of maxima or minima passing one point per unit time
P	true power spectral density function (in^2 / cps)

P_a	average measured power spectral density (in^2/cps)
P_c	calculated average power spectral density (in^2/cps)
P_o	spectral density at output of band-pass filter $\left Y(f, f_M) \right ^2 P(f)$ (in^2/cps)
$\Delta P / \Delta L$	pressure gradient ($\text{lb} / \text{ft}^2 / \text{ft}$)
Q_o	oil flow rate (U.S. gal/min)
Q_w	water flow rate (U.S. gal/min)
Re_o	superficial oil Reynolds number based on total height of conduit
Re_w	superficial water Reynolds number based on total height of conduit
s	deviation of calculated power spectrum from actual power spectrum ($P_c(f_M) - P_a(f_M)$)
S	total standard deviation defined as $\sqrt{\frac{s^2}{44}}$
t, T	time (sec)
$\bar{v}$	mean velocity of water phase (ft/sec)
V	photomultiplier output (mv)
V_o	photomultiplier output voltage with nothing in path of light (mv)
$\bar{V}$	time average output voltage from photomultiplier (mv)
V'	perturbation output voltage (mv)
V_r'	root-mean-square perturbation output voltage (mv)
(VN)	output voltage due to noise (mv)
X	some random signal
X'	perturbation random signal

$\overline{X}$	time average of random signal
X_r'	root-mean-square perturbation random signal
Y	transfer function of band-pass filter

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APPENDICES

APPENDIX A

CALIBRATION OF DYE SENSITIVITY

The applicability of the Beer-Lambert law of light absorption to the effect of dye concentration and liquid height was confirmed for each dye concentration used, as shown in Figure 7, using the calibration cell illustrated in Figure 11. The calibration cell was a "Plexiglas" cylinder, 4.0 inches in diameter and 4.0 inches deep. The plunger was raised and lowered by means of screw threads having a pitch of 0.125 inches. When the cell had been charged with the proper quantity of dyed oil to be tested, the full height to which the plunger could be raised and still have a very small portion of its bottom in the fluid was 1 inch. When the plunger was fully depressed there was no spill-off of fluid through the vent holes at the top. The cell was highly polished to eliminate losses of light due to refraction from rough surfaces.

Oil solutions, colored with Sudan Red 4BA dye, were prepared over a range of dye concentrations. The concentrations were determined by measuring the per cent light transmission at 530 m μ with a Bausch and Lomb Spectronic 20 Colorimeter and varied over a range corresponding to 9.2 per cent to 83.5 per cent transmission.

The effect of the oil film thickness on the voltage output from the photomultiplier tube for the various solutions was measured using the calibration cell. The cell was filled with oil to the required level and the plunger depressed to the bottom to give a zero height reading. After depressing the plunger any excess oil was carefully wiped from the top of the cell. The base reading V_o was obtained by placing the cell with the plunger fully depressed into the light path and noting the voltage reading on the digital voltmeter. This voltage was usually kept around 1265 mv to assure the same output intensity for each calibration run. After obtaining this base voltage, the plunger was raised 1 turn, or 0.125 inches, and a voltage reading obtained. This procedure was repeated until the plunger had been turned 8 times, which corresponded to a rise of 1 inch. The plunger was then lowered in a like manner until it reached zero height. If the base voltage had changed during this calibration interval, it was reset and the procedure repeated until the base voltage had remained constant throughout an entire run.

According to the Beer-Lambert law (Equation 20)

$$\ln \frac{V}{V_o} = -k h \quad (36)$$

at a constant dye concentration, a plot of $\ln \frac{V}{V_o}$ versus h should result in a straight line with a slope of $(-k)$. This, in fact, was

found to be the case and is illustrated in Figure 7, confirming the Beer-Lambert relationships.

A crossplot of the per cent transmission at 530 $m\mu$ obtained by the colorimeter versus $-\frac{\ln \frac{V}{V_0}}{h}$, the negative of the slope obtained from Figure 7, resulted in the straight line shown in Figure 8. This graph was used to determine k for the various dye concentrations encountered in the actual experiment.

APPENDIX B

DESCRIPTION OF COMPUTER PROGRAM

This section describes in more detail the computer solution of the power spectral density equation

$$P_a(f_M) = \int_0^\infty H(f, f_M) P(f) df \quad (33)$$

which was outlined earlier in the section on the Treatment of Data.

The program is naturally begun by reading in the data. As mentioned previously, the experimental smoothed values of the output voltage, along with the contribution due to noise, are read in for each whole number frequency from 1 to 100. Along with these values, the values of the power transfer function $H(f, f_M)$ from $(f_M) = 0.2$ to $(f_M) = 2.2$ are read in. Eleven such values are included which allows for 10 equal intervals for the ensuing numerical work. For each power spectrum the value for $k\bar{V}$ is read in to enable $P_a(f_M)$ to be calculated from Equation (37). As discussed in the section dealing with the treatment of noise, it was deemed more accurate to convert the voltage due to noise to its equivalent perturbation height before subtracting it from the raw perturbation height to give the true root-mean-square perturbation height. This conversion to height equivalent was done prior to running the program

and these were the values read into the computer for the noise contribution. The values for $P_a(f_M)$ were then calculated by

$$P_a(f_M) = \left[\frac{V_r^i(f_M)}{k \bar{V}} - (hN)_r^i(f_M) \right]^2 \quad (37)$$

where

$$(hN)_r^i(f_M) = \frac{(VN)_r^i(f_M)}{(\bar{VN})} \quad (38)$$

Equation (37) follows from Equation (30).

An initial assumption was then made for $P(f)$. It was decided to use the values of $P_a(f_M)$ for the initially assumed values of $P(f)$. The integral was then calculated at whole number values of frequency f_M , that is for $f_M = 1, 2, 3 \dots 45$. A linear interpolation formula was used to obtain values of $P(f)$ if they happened not to fall on an already known value. Thus 11 values of the transfer function along with 99 values of the power spectrum were utilized. The 99 values appear because when $f_M = 45$, the upper limit of f is $45 \times 2.2 = 99$.

The values of $P_c(f_M)$ were determined in the following manner for $i = 1, 2, 3 \dots 45$:

- (1) The curves for the transfer function plus the power spectral density were broken into 10 equal intervals, that is,

$$f_j = \frac{f_{M_i}}{5} + f_{j-1} \quad \begin{matrix} 1 \leq i \leq 45 \\ 2 \leq j \leq 11 \end{matrix} \quad (39)$$

where

$$f_{M_i} = f_{M_{i-1}} + 1 \quad (40)$$

and

$$f_1 = f_{M_i}/5 \quad (41)$$

- (2) A straight line was placed between the 2 whole number values of frequency in which range f_j was contained. The value of $P(f_j)$ was then determined by linear interpolation.

- (3) A straight line was assumed for each curve over each interval j . That is,

$$H(f, f_{M_i}) = A_j f + B_j \quad (42)$$

$$P(f) = C_j f + D_j \quad (43)$$

- (4) The values of the slopes A and C plus the values of the intercepts B and D are evaluated for each interval, j .

$$A_j = \frac{H(f_j, f_{M_i}) - H(f_{j-1}, f_{M_i})}{f_j - f_{j-1}} \quad 2 \leq j \leq 11 \quad (44)$$

$$B_j = H(f_j, f_{M_i}) - A_j f_j \quad 2 \leq j \leq 11 \quad (45)$$

$$C_j = \frac{P(f_j) - P(f_{j-1})}{f_j - f_{j-1}} \quad 2 \leq j \leq 11 \quad (46)$$

$$D_j = P(f_j) - C_j f_j \quad 2 \leq j \leq 11 \quad (47)$$

(5) These two straight line assumptions were multiplied and integrated analytically.

$$P_c(f_{M_i}) = \sum_{j=2}^{11} \int_{f_{j-1}}^{f_j} (A_j f + B_j) (C_j f + D_j) df \quad (48)$$

$$= \sum_{j=2}^{11} \left[A_j C_j (f_j^3 - f_{j-1}^3) / 3 + \right. \\ \left. (B_j C_j + A_j D_j) (f_j^2 - f_{j-1}^2) / 2 + \right. \\ \left. B_j D_j (f_j - f_{j-1}) \right] \quad (49)$$

As is illustrated here, a second order approximation over each interval j results for the curve obtained by multiplying together $H(f, f_M)$ and $P(f)$. This method was found to be much more accurate than the method of assuming a constant, mid-interval value for both $H(f, f_M)$ plus $P(f)$ at each interval and then carrying out the integration.

(6) The value for i was then increased by 1 until i was equal to 45. This gave 45 values of $P_c(f_M)$ for $f_M = 1, 2, 3 \dots 45$.

(7) At these frequencies the deviation s was determined where

$$s = P_c(f_M) - P_a(f_M) \quad (50)$$

The total variance S^2 was also determined where

$$S^2 = \frac{s^2}{44} \quad (51)$$

The variance was used primarily as a parameter with which successive iterations could be compared to see if the solution was converging. It was used to compare various methods and initial assumptions tried.

(8) After determining $P_c(f_M)$ it was necessary to modify the assumed curve for $P(f)$ such that the next values obtained for $P_c(f_M)$ would more closely approximate $P_a(f_M)$. These corrections were applied utilizing a geometric correction at the points where $P_c(f_M)$ was determined (Equation 35).

(9) This newly assumed curve for the power spectral density was then operated on in an identical manner to the aforementioned procedure. This iterative procedure

was terminated at the end of 10 iterations at which time the changes in the values obtained for $P_c(f_M)$ became essentially insignificant.

- (10) The total power under the power spectral density curve was determined by integrating the final value of $P(f)$ numerically utilizing Simpson's rule⁽²¹⁾. In applying Simpson's rule the power spectral density curve was divided into 52 intervals, each with a width of 1 cps. The integrals required to determine the number of times the surface crosses the mean height per second $N(\bar{h})$ from Equation (28), with the integrals required to solve for $N(h_M)$ from Equation (29) were likewise calculated via Simpson's rule.

A method similar to this was employed which found $P_c(f_M)$ only at every fifth value of f_M . Using this method, only values of $P(f)$ at every whole value of frequency were utilized which eliminated the necessity of interpolation for values not at whole number values of frequency. It necessitated, however, the interpolation for a correction factor to apply at frequencies which were not multiples of five. The correction applied was the following:

$$P_{\text{new}}(f) = P_{\text{old}}(f) \left[\frac{P_a(f_M - 5)}{P_c(f_M - 5)} + \frac{[f - (f_M - 5)]}{5} \cdot \left(\frac{P_a(f_M)}{P_c(f_M)} - \frac{P_a(f_M - 5)}{P_c(f_M - 5)} \right) \right] \quad (52)$$

for $f_M = 5, 10, 15 \dots 45$. This method also yielded a continuous curve but the total power did not agree with the measured total power as well as the total power for the previously described method. It was discovered also that the frequencies at which $P_c(f_M)$ was obtained were weighted more heavily than the other frequencies and any maxima or minima which were present appeared only at these frequencies.

In order to test the effect of the width of the pass-band the same technique was applied to calculate the power spectral density function. The only difference was that the appropriate value for n was placed into the equation for the transfer functions evolved for both widths tried. The values of $P(f)$ were then found for mid-band frequencies f_M from 1 cps to 45 cps.

Table 1

SUMMARY OF AVERAGE PROPERTIES

Run No.	Re _O	Re _w	$\bar{v}_w$ (ft/sec)	$\frac{\Delta P}{\Delta L}$ (lb/ft ² /ft)	$\bar{h}$ (in.)	$h_T' \times 10^3$ (inches)	meas.	Input ratio Q_w/Q_O	Insitu ratio $\bar{h}_w/\bar{h}_O$
1	1532	4, 436	0.958	.0833	.708	3.67		.442	.413
2	1532	10, 740	1.303	.3007	.491	12.1		1.05	1.04
3	3107	4, 436	1.263	.4007	.778	22.9		.218	.286
4	3107	10, 740	1.464	.7200	.547	30.5		.517	.828
5	3107	17, 810	1.732	-	.353	39.6		.875	1.83
6	3107	19, 110	1.834	-	.345	49.8		.937	1.90
6A	294	4, 436	0.529	.0469	.471	-		2.30	1.12
7	294	10, 740	0.904	.1326	.266	5.9		5.47	2.76
8	294	17, 810	1.379	.2715	.186	23.5		9.25	4.37
9	294	19, 110	1.458	.3073	.175	35.3		9.92	4.72
10	294	20, 970	1.567	.3565	.158	34.4		10.9	5.33
11	105	10, 740	0.761	.1130	.139	11.5		15.3	6.19
12	105	12, 920	0.915	.1550	.110	26.2		18.8	8.09
13	105	17, 810	1.223	.2834	.083	58.6		25.6	11.1
13A	105	19, 110	1.274	.3471	.056	-		27.8	16.9

Table 2

POWER SPECTRUM OF RUN 1

$$Re_o = 1532$$

$$Re_w = 4436$$

f_M (cps)	V (mv)	$P_a(f_M) \times 10^6$ (in ² /cps)	$P(f) \times 10^6$ (in ² /cps)	$P_c(f_M) \times 10^6$ (in ² /cps)
2.0	.320	.368	.764	.358
3.0	.340	.526	.618	.526
4.0	.405	.727	.546	.733
5.0	.470	1.033	.820	1.019
6.0	.530	1.279	.744	1.287
7.0	.510	1.457	.750	1.480
8.0	.540	1.596	1.027	1.592
9.0	.570	1.632	1.215	1.610
10.0	.510	1.303	.573	1.294
11.0	.490	1.023	.151	1.113
12.0	.445	.871	.123	.935
14.0	.400	.618	.146	.723
16.0	.360	.410	.0357	.440
18.0	.290	.282	.0425	.269
20.0	.262	.173	.0166	.150
22.0	.232	.0979	.0003	.151
24.0	.210	.0566	.0002	.117
26.0	.196	.0330	.0001	.0489
28.0	.180	.0213		.0230
30.0	.169	.0126		.0303
35.0	.161	.0072		.0140
42.0	.160	.0047		.0055
50.0	.160	.0017		

$$S \times 10^6 = .0348$$

Table 3

POWER SPECTRUM OF RUN 1A

$$(f_2/f_1) = 1.2$$

$$Re_o = 1532$$

$$Re_w = 4436$$

f_M (cps)	V (mv)	$P_a(f_M) \times 10^6$ (in ² /cps)	$P(f) \times 10^6$ (in ² /cps)	$P_c(f_M) \times 10^6$ (in ² /cps)
1.7	.280			
2.0		.411	.205	.414
2.2	.310			
2.5	.450			
3.0	.550	1.383	.869	1.389
3.5	.590			
4.0		2.407	1.317	2.382
4.3	.660			
5.0		3.162	1.037	3.198
5.1	.745			
6.0		3.856	1.212	3.893
6.1	.800			
7.0		4.446	1.711	4.424
7.4	.870			
8.0		4.365	1.529	4.313
8.9	.820			
9.0		3.836	.815	3.851
10.7	.745			
11.0		3.050	.438	3.087
12.7	.710			
13.0		2.487	.341	2.491
15.0		1.974	.307	1.942
15.3	.635			
18.0		1.207	.100	1.218
18.4	.500			
22.0	.322	.336	.00013	.596
26.4	.245			
31.6	.213			
38.0	.175			
45.6	.175			

$$S \times 10^6 = .0935$$

Table 4

POWER SPECTRUM OF RUN 1B

$$(f_2/f_1 = 1.5)$$

$$Re_o = 1532$$

$$Re_w = 4436$$

f_M (cps)	V (mv)	$P_a(f_M) \times 10^6$ (in ² /cps)	$P(f) \times 10^6$ (in ² /cps)	$P_c(f_M) \times 10^6$ (in ² /cps)
2.0		2.311	1.189	2.313
2.4	.720			
3.0		3.480	1.143	3.487
3.2	.800			
5.0		5.682	1.013	5.688
5.4	1.00			
8.0		7.704	1.359	7.632
8.1	1.10			
12.0		4.879	.393	4.813
12.1	.900			
18.0		2.095	.086	2.031
18.1	.620			
22.0		1.164	.026	1.199
26.0		.682	.014	.644
27.3	.370			
40.9	.225			

$$S \times 10^6 = .0875$$

Table 5

POWER SPECTRUM OF RUN 2

$$Re_o = 1532$$

$$Re_w = 10,470$$

f_M (cps)	V (mv)	$P_a(f_M) \times 10^6$ (in ² /cps)	$P(f) \times 10^6$ (in ² /cps)	$P_c(f_M) \times 10^6$ (in ² /cps)
2.0	.72	1.608	3.517	1.607
4.0	.78	2.346	2.029	2.346
5.0		2.816	2.089	2.808
6.0	1.00	3.275	1.935	3.291
7.0		3.760	2.037	3.763
8.0	1.12	4.233	2.081	4.219
9.0		4.608	1.819	4.610
10.0	1.17	5.067	1.672	5.088
11.0		5.593	1.776	5.559
12.0	1.35	6.143	1.876	6.072
13.0		6.686	1.835	6.645
14.0	1.43	7.127	1.732	7.183
15.0		7.512	1.545	7.713
16.0	1.45	7.871	1.689	8.076
17.0		8.239	1.957	8.346
18.0	1.53	8.668	2.503	8.651
19.0		8.603	2.695	8.475
20.0	1.53	8.591	2.699	8.432
21.0		8.080	2.307	7.932
22.0	1.43	7.542	1.751	7.447
24.0	1.37	6.441	.930	6.595
25.0		5.931	.682	6.137
26.0	1.15	5.340	.522	5.561
27.5	1.16			
28.0	1.15	4.315	.362	4.354
30.0	1.10	3.463	.190	3.470
32.0	.96	2.896	.122	2.991
34.0	.92	2.380	.069	2.595
36.0	.835	1.923	.055	2.071
38.0	.740	1.534	.048	1.513

40.0	.700	1.240	.034	1.150
42.0	.645	.999	.023	.888
44.0	.610	.802	.0107	.751
46.0	.560	.637	.0052	
48.0	.545	.556	.0046	
49.0	.500	.424	.0035	
50.0	.465	.366	.0030	
52.0	.450	.267	.0022	
54.0	.420	.197	.0016	

$$S \times 10^6 = .1618$$

Table 6

POWER SPECTRUM OF RUN 3

$$Re_o = 3107$$

$$Re_w = 4436$$

f_M (cps)	V (mv)	$P_a(f_M) \times 10^6$ (in ² /cps)	$P(f) \times 10^6$ (in ² /cps)	$P_c(f_M) \times 10^6$ (in ² /cps)
2	1.30	14.86	32.44	14.85
3	1.55	18.39	20.87	18.42
4	1.60	22.71	19.95	22.65
6	1.95	33.71	21.94	33.62
8	2.20	41.85	18.97	41.74
10	2.30	47.82	16.17	48.85
12	2.43	52.14	24.03	51.45
14	2.22	45.77	14.06	45.34
16	2.07	38.96	5.451	41.29
18	2.00	35.57	6.315	35.43
20	1.95	32.83	5.587	32.22
22	1.85	30.37	4.053	31.13
24	1.80	28.30	3.949	28.81
26	1.70	26.47	4.299	25.65
28	1.75	24.87	3.702	24.02
30	1.70	23.39	2.710	23.32
32	1.55	22.07	2.080	23.19
34	1.50	20.73	1.926	21.66
36	1.60	19.53	1.940	19.68
38	1.45	18.33	1.936	17.81
40	1.55	17.17	1.677	16.43
42	1.50	16.08	1.343	15.70
44	1.50	15.02	1.083	15.03
46	1.40	13.97	0.930	
48	1.60	13.00	0.866	
50	1.25	11.89	0.792	
52	1.18	10.86	0.723	
54	1.15	9.89	0.659	

$$S \times 10^6 = 1.312$$

Table 7

POWER SPECTRUM OF RUN 4

$$Re_o = 3107$$

$$Re_w = 10,740$$

f_M (cps)	V (mv)	$P_a(f_M) \times 10^6$ (in ² /cps)	$P(f) \times 10^6$ (in ² /cps)	$P_c(f_M) \times 10^6$ (in ² /cps)
2.0	2.16	20.69	44.73	20.67
3.0	2.16	25.97	30.26	26.02
4.0	2.34	30.77	28.28	30.74
6.0	2.96	38.82	23.63	38.90
8.0	3.08	45.38	21.68	45.20
10.0	3.28	49.58	17.36	49.86
12.0	3.36	53.03	17.58	53.06
14.0	3.38	53.49	16.64	53.36
16.0	3.19	50.46	13.23	50.44
18.0	3.10	46.54	10.50	46.42
20.0	3.06	41.01	7.30	40.98
22.0	3.01	36.22	5.08	36.55
24.0	2.75	32.20	3.92	32.69
26.0	2.45	28.76	3.37	28.77
28.0	2.36	26.18	3.10	26.00
30.0	2.28	23.67	2.59	23.47
32.0	2.21	21.48	2.28	21.49
34.0	2.11	19.40	1.94	19.32
36.0	1.99	17.58	1.69	17.53
38.0	1.88	15.81	1.48	15.59
40.0	1.71	14.19	1.14	14.02
42.0	1.83	12.67	.890	12.66
44.0	1.73	11.20	.655	11.27
46.0	1.63	9.93	.521	
48.0	1.57	8.63	.453	
50.0	1.25	7.36	.386	
52.0	1.23	6.34	.333	
54.0	1.22	5.52	.289	

$$S \times 10^6 = 1.430$$

Table 8

POWER SPECTRUM OF RUN 5

$$Re_o = 3107$$

$$Re_w = 17,810$$

f_M (cps)	V (mv)	$P_a(f_M) \times 10^6$ (in ² /cps)	$P(f) \times 10^6$ (in ² /cps)	$P_c(f_M) \times 10^6$ (in ² /cps)
2.0	2.90	23.90	49.53	23.89
3.0	3.45	33.06	40.19	33.10
4.0	3.75	39.87	38.00	39.84
6.0	4.05	47.87	29.70	47.86
8.0	4.25	53.50	24.42	53.55
10.0	4.45	58.76	21.60	58.77
12.0	4.78	63.86	19.68	68.68
14.0	4.95	68.58	17.84	68.45
16.0	4.95	72.41	15.95	72.81
18.0	4.90	76.49	16.01	76.95
20.0	5.20	78.47	16.72	78.35
22.0	5.22	79.27	16.68	78.60
24.0	5.12	75.96	14.00	75.46
26.0	5.00	71.82	10.65	72.20
28.0	4.84	67.50	8.32	68.39
30.0	4.72	63.60	7.31	64.30
32.0	4.56	59.68	6.82	59.32
34.0	4.44	56.02	6.17	55.45
36.0	4.32	52.32	5.27	51.78
38.0	4.20	48.75	4.31	48.60
40.0	4.00	45.30	3.51	45.57
42.0	3.90	42.20	2.99	42.52
44.0	3.80	39.11	2.52	39.34
48.0	3.52	33.80	2.15	
50.0	3.33	31.29	2.00	
52.0	3.25	28.93	1.85	
54.0	3.20	26.85	1.72	

$$S \times 10^6 = 1.351$$

Table 9

POWER SPECTRUM OF RUN 6

$$Re_o = 3107$$

$$Re_w = 19,110$$

f_M (cps)	V (mv)	$P_a(f_M) \times 10^6$ (in ² /cps)	$P(f) \times 10^6$ (in ² /cps)	$P_c(f_M) \times 10^6$ (in ² /cps)
2.0	3.90	44.83	96.36	44.80
3.0	4.50	56.81	67.10	56.92
4.0	5.40	66.54	61.81	66.48
8.0	5.65	92.69	43.21	92.68
12.0	6.25	112.41	34.98	111.92
16.0	6.65	127.65	28.79	128.78
18.0	6.80	134.21	31.97	133.76
20.0	6.85	136.19	31.72	134.83
22.0	6.65	129.54	24.69	129.14
24.0	6.50	123.01	18.86	124.60
26.0	6.40	115.89	15.97	117.27
28.0	6.15	108.43	15.10	108.30
30.0	6.05	100.52	13.04	99.63
32.0	5.80	92.57	10.96	92.07
34.0	5.55	84.79	8.76	84.92
36.0	5.20	77.18	7.29	77.72
38.0	4.95	69.91	6.11	69.81
40.0	4.75	62.86	4.73	62.49
42.0	4.55	56.06	3.59	55.47
44.0	4.25	50.26	2.59	49.85
46.0	4.10	46.44	2.22	
48.0	4.10	43.13	2.07	
50.0	4.02	39.98	1.92	
52.0	3.90	37.34	1.79	
54.0	3.60	34.68	1.66	

$$S \times 10^6 = 3.105$$

Table 10

POWER SPECTRUM OF RUN 7

$$Re_o = 294$$

$$Re_w = 10,740$$

f_M (cps)	V (mv)	$P_a(f_M) \times 10^6$ (in ² /cps)	$P(f) \times 10^6$ (in ² /cps)	$P_c(f_M) \times 10^6$ (in ² /cps)
2.0	0.690	0.769	1.227	.769
2.5	0.800			
3.0	0.940	1.721	2.112	1.724
4.0	0.990	2.677	2.592	2.667
5.0	1.24	3.478	2.499	3.480
6.0	1.36	4.122	2.597	4.137
7.0	1.43	4.516	2.700	4.533
8.0	1.42	4.643	2.909	4.620
9.0	1.44	4.361	2.463	4.319
10.0	1.30	3.700	1.355	3.719
11.0	1.19	3.157	0.834	3.223
12.0	1.18	2.679	0.625	2.705
13.0	1.16	2.230	0.450	2.224
14.0	1.05	1.910	0.343	1.906
15.0	0.950	1.623	0.223	1.667
16.0	0.870	1.360	0.196	1.371
17.0	0.800	1.138	0.162	1.118
18.0	0.790	1.017	0.170	0.974
19.0	0.728	0.830	0.116	0.781
20.0	0.606	0.722	0.087	0.702
21.0	0.665	0.591	0.052	0.594
22.0	0.645	0.507	0.032	0.538
24.0	0.545	0.341	0.011	0.378
26.0	0.482	0.229	0.0041	0.247
28.0	0.453	0.156	0.0015	0.171
30.0	0.404	0.109		
32.0	0.372	0.072		
34.0	0.350	0.048		
36.0	0.320	0.030		
38.0	0.295	0.018		
42.0	0.277	0.012		
46.0	0.300	0.017		
50.0	0.305	0.015		
54.0	0.317	0.007		

$$S \times 10^6 = .0276$$

Table 11

POWER SPECTRUM OF RUN 8

$$Re_o = 294$$

$$Re_w = 17,810$$

f_M (cps)	V (mv)	$P_a(f_M) \times 10^6$ (in ² /cps)	$P(f) \times 10^6$ (in ² /cps)	$P_c(f_M) \times 10^6$ (in ² /cps)
2.0	1.26	2.74	5.26	2.83
2.5	1.53			
2.9	1.72			
3.4	2.12			
4.0	2.54	13.59	1.07	15.67
4.4	3.10			
5.0	4.42	38.87	14.73	40.46
5.5	4.86			
6.0	5.37	65.10	50.85	65.37
6.5	5.83			
7.0	5.95	79.10	83.79	78.54
7.5	5.70			
8.0	5.39	65.28	43.98	64.54
8.5	5.35			
9.0	4.86	50.23	9.81	53.73
10.0	4.37	40.53	7.63	42.24
11.0	3.98	33.39	6.16	32.81
12.0	3.83	28.57	3.16	29.26
13.0	3.43	25.20	2.68	26.19
14.0	3.32	22.91	2.65	23.10
15.0	3.18	21.73	3.48	20.39
16.0	3.27	22.31	2.98	21.79
17.0	3.34	23.63	3.16	24.14
18.0	3.43	25.51	4.20	26.14
19.0	3.53	26.40	5.54	27.20
20.0	3.62	27.52	7.37	27.56
21.0	3.60	27.51	8.05	26.99
22.0	3.59	27.18	7.88	26.46
24.0	3.50	25.48	5.69	25.00
25.0	3.41	24.20	4.19	24.44
26.0	3.30	22.94	3.32	23.73

28.0	3.13	20.48	2.34	21.41
30.0	2.98	18.15	1.93	18.33
32.0	2.82	16.08	1.59	15.59
34.0	2.68	14.27	1.14	14.08
36.0	2.62	12.44	0.733	12.87
38.0	2.45	10.94	0.535	11.77
40.0	2.21	9.53	0.420	10.17
42.0	2.14	8.39	0.388	8.43
44.0	1.98	7.50	0.389	7.16
46.0	1.91	6.74	0.348	
50.0	1.83	5.50	0.274	
54.0	1.72	4.69	0.242	

$$S \times 10^6 = .914$$

Table 12

POWER SPECTRUM OF RUN 9

$$Re_o = 294$$

$$Re_w = 19,110$$

f_M (cps)	V (mv)	$P_a(f_M) \times 10^6$ (in ² /cps)	$P(f) \times 10^6$ (in ² /cps)	$P_c(f_M) \times 10^6$ (in ² /cps)
2.0	1.60	4.92	9.35	5.03
2.2	1.74			
2.4	1.82			
2.8	1.93			
3.0		8.02	12.25	7.76
3.2	2.09			
3.6	2.53			
4.2	3.12			
4.8	3.95			
5.4	4.93			
6.0	6.35	91.48	47.36	93.35
6.5	7.25			
7.0	7.55	129.95	113.04	129.64
7.5	7.70			
8.0	7.75	137.22	125.01	135.78
8.5	7.60			
9.0	7.20	117.57	68.77	116.63
9.5	6.75			
10.0	6.20	87.33	16.57	92.89
11.0	5.40	66.70	6.46	73.69
12.0	5.00	51.92	3.60	55.92
13.0	4.46	43.97	3.31	45.32
14.0	4.27	38.89	2.56	40.39
15.0	4.11	34.27	2.82	35.03
16.0	3.74	31.25	2.05	32.28
17.0	3.72	29.19	1.91	29.96
18.0	3.72	29.46	2.48	30.09
19.0	4.02	34.60	6.48	34.48
20.0	4.15	37.38	10.18	37.01

21.0	4.19	37.75	11.84	37.04
22.0	4.08	36.61	11.47	35.87
24.0	3.96	33.36	6.81	32.87
28.0	3.84	28.57	2.82	30.18
30.0	3.62	26.75	3.57	26.65
32.0	3.52	25.16	3.17	24.04
36.0	3.27	22.83	1.41	23.95
42.0	3.19	20.00	1.61	20.39
48.0	3.01	17.50	1.56	
54.0	2.79	14.80	1.32	

$$S \times 10^6 = 2.080$$

Table 13

POWER SPECTRUM OF RUN 10

$$Re_o = 294$$

$$Re_w = 20,970$$

f_M (cps)	V (mv)	$P_a(f_M) \times 10^6$ (in ² /cps)	$P(f) \times 10^6$ (in ² /cps)	$P_c(f_M) \times 10^6$ (in ² /cps)
2.0	1.38	3.26	5.25	3.23
2.4	1.59			
3.0	1.82	5.97	9.02	6.19
3.6	2.00			
4.2	2.31			
4.8	2.82			
5.4	3.65			
6.0	4.61	40.02	3.13	46.03
6.5	5.22			
7.0	5.83	71.78	6.90	86.13
7.5	6.70			
8.0	7.41	120.47	69.06	121.72
8.5	8.67			
9.0	8.73	163.01	176.58	161.26
9.5	8.62			
10.0	8.25	136.42	110.63	134.59
11.0	7.05	100.36	12.87	111.04
12.0	6.61	79.38	3.11	103.97
13.0	5.62	66.34	4.47	74.72
14.0	5.27	57.39	8.70	54.10
15.0	5.01	51.63	1.65	62.46
16.0	4.82	47.40	1.53	60.27
17.0	4.61	43.95	2.05	53.52
18.0	4.56	42.12	6.91	41.53
19.0	4.43	40.25	12.32	36.88
20.0	4.54	41.58	14.22	38.71
21.0	4.57	42.15	12.03	39.50
22.0	4.58	42.38	8.23	41.30
23.0	4.46	40.51	4.67	44.18
24.0	4.42	38.77	2.91	47.18
25.0	4.33	37.60	2.25	46.90

26.0	4.32	37.25	2.93	43.04
27.0	4.30	36.96	4.22	38.46
28.0	4.34	37.86	6.14	36.53
29.0	4.48	39.15	7.80	36.83
30.0	4.39	38.58	8.16	36.36
32.0	4.20	35.64	5.91	34.80
36.0	4.10	32.78	2.19	36.59
42.0	3.95	29.64	2.28	29.02
48.0	3.80	27.61	2.24	
54.0	3.69	25.18	2.04	

$$S \times 10^6 = 6.28$$

Table 14

POWER SPECTRUM OF RUN 11

$$\text{Re}_o = 105$$

$$\text{Re}_w = 10,740$$

f_M (cps)	V (mv)	$P_a(f_M) \times 10^6$ (in ² /cps)	$P(f) \times 10^6$ (in ² /cps)	$P_c(f_M) \times 10^6$ (in ² /cps)
2.0	0.750	0.963	1.96	.964
2.4	0.830			
2.6	0.842			
2.8	0.880			
3.0		1.44	1.69	1.46
3.2	1.06			
3.6	1.11			
4.0		2.60	1.86	2.49
4.2	1.47			
4.8	1.71			
5.4	1.90			
6.0	2.36	9.63	3.97	9.75
6.5	2.59			
7.0	2.84	14.68	8.23	14.81
7.5	2.86			
8.0	2.99	17.87	11.61	17.86
8.5	3.12			
9.0	3.18	19.99	14.42	19.81
9.5	3.10			
10.0	2.90	17.87	9.89	17.73
11.0	2.82	15.53	5.17	15.73
12.0	2.66	13.24	3.49	13.50
13.0	2.42	10.69	2.39	10.74
14.0	2.09	8.24	1.23	8.17
15.0	1.93	6.29	0.304	6.96
16.0	1.68	4.67	0.100	5.69
17.0	1.52	3.57	0.054	4.42
18.0	1.33	2.78	0.051	3.07
19.0	1.15	1.98	0.020	2.12
20.0	0.980	1.52	0.008	1.76

22.0	0.842	0.880	0.001	1.48
24.0	0.730	0.535		
26.0	0.630	0.343		
28.0	0.542	0.221		
30.0	0.520	0.155		
32.0	0.444	0.110		
36.0	0.405	0.055		
42.0	0.356	0.024		
48.0	0.348	0.015		
54.0	0.328	0.003		

$$S \times 10^6 = .299$$

Table 15

POWER SPECTRUM OF RUN 12

$$Re_o = 105$$

$$Re_w = 12,920$$

f_M (cps)	V (mv)	$P_a(f_M) \times 10^6$ (in ² /cps)	$P(f) \times 10^6$ (in ² /cps)	$P_c(f_M) \times 10^6$ (in ² /cps)
2.0	4.02	23.39	4.73	27.49
2.1	4.45			
2.2	4.40			
2.3	4.88			
2.4	5.52			
2.5	5.60			
2.6	5.72			
2.7	5.87			
2.8	6.08			
2.9	6.11			
3.0	6.22	77.28	197.09	77.12
3.2	5.85			
3.4	5.21			
3.6	4.59			
3.8	4.04			
4.0		22.75	0.169	39.93
4.2	3.40			
4.8	2.62			
5.0		11.36	0.070	18.23
5.4	2.35			
6.0	2.44	10.75	0.315	13.94
6.5	2.56			
7.0	2.64	13.21	4.88	12.84
7.5	2.86			
8.0	3.04	17.32	5.71	17.51
8.5	3.22			
9.0	3.43	21.65	9.62	21.78
9.5	3.48			
10.0	3.59	23.77	12.33	23.51
11.0	3.61	25.06	10.58	25.13
12.0	3.72	25.34	10.01	25.71

13.0	3.63	25.18	11.18	25.19
14.0	3.62	23.59	10.56	23.21
15.0	3.43	20.44	6.85	19.93
16.0	2.97	16.20	2.42	16.54
17.0	2.80	13.40	.772	15.55
18.0	2.59	11.52	.481	13.77
19.0	2.41	9.46	.377	10.82
20.0	2.23	8.25	.693	7.92
21.0	2.15	7.19	.514	6.84
22.0	2.09	6.36	.289	6.47
24.0	1.89	5.05	.128	6.06
26.0	1.46	4.25	.231	4.53
28.0	1.33	3.55	.523	3.16
29.0	1.31	3.39	.581	3.00
30.0	1.38	3.17	.532	2.82
32.0	1.33	2.77	.291	2.54
34.0	1.29	2.48	.119	2.83
38.0	1.23	2.10	.106	2.49
42.0	1.15	1.80	.194	1.70
48.0	1.10	1.46	.136	
54.0	1.03	1.08	.101	

$$S \times 10^6 = 3.01$$

Table 16

POWER SPECTRUM OF RUN 13

$$Re_o = 105$$

$$Re_w = 17,810$$

f_M (cps)	V (mv)	$P_a(f_M) \times 10^6$ (in ² /cps)	$P(f) \times 10^6$ (in ² /cps)	$P_c(f_M) \times 10^6$ (in ² /cps)
2.0	2.38	8.76	.490	9.08
2.4	3.43			
2.8	4.78			
3.0		78.35	10.14	97.85
3.2	6.10			
3.6	8.20			
4.0	10.3	228.26	412.70	225.97
4.4	11.0			
4.6	11.2			
4.8	11.8			
5.0	11.2	205.82	130.53	214.11
5.4	10.6			
6.0	9.80	171.58	131.15	167.56
6.5	8.80			
7.0	7.61	116.27	20.75	125.03
8.0	7.32	89.14	22.35	102.38
8.5	7.20			
9.0	7.06	89.12	38.70	84.86
9.5	7.03			
10.0	6.95	83.04	22.64	84.89
11.0	6.46	77.82	20.71	80.99
12.0	6.58	73.95	25.39	72.38
13.0	6.62	71.58	22.98	68.35
14.0	6.64	69.70	14.92	69.12
15.0	6.35	68.54	8.82	75.44
16.0	6.19	68.98	9.17	74.93
17.0	6.44	74.11	14.04	75.71
18.0	6.59	76.70	18.34	75.27
19.0	6.68	78.19	20.09	75.62
20.0	6.67	79.43	19.98	77.81

21.0	6.64	79.64	17.73	79.15
22.0	6.68	79.13	15.28	80.18
24.0	6.60	76.50	12.82	77.85
26.0	6.39	72.97	11.31	72.49
28.0	6.28	69.43	9.66	67.94
30.0	6.17	66.89	7.71	67.09
32.0	6.12	62.42	5.85	64.00
34.0	5.70	60.22	5.00	62.33
38.0	5.75	56.63	4.92	55.78
42.0	5.60	54.35	4.50	53.17
46.0	5.56	52.32	3.64	
48.0	5.62	51.69	3.59	
54.0	5.55	48.83	3.39	

$$S \times 10^6 = 4.10$$

Table 17

COMPARISON OF SPECTRAL DENSITIES BY
DIFFERENT CALCULATIONAL PROCEDURES

Run 2

$$\text{Re}_O = 1537$$

$$\text{Re}_W = 10,740$$

f_M (cps)	$P_a(f_M) \times 10^6$ (in ² /cps)	$P(f)^* \times 10^6$ (in ² /cps)	$P_c^*(f_M) \times 10^6$ (in ² /cps)	$P_c(f_M) \times 10^{6**}$ (in ² /cps)
5.0	2.816	2.083	3.218	2.808
10.0	5.067	1.874	5.236	5.088
15.0	7.512	1.852	7.156	7.713
20.0	8.591	1.589	7.420	8.432
25.0	5.931	.877	6.180	6.137
30.0	3.463	.427	4.587	3.470
35.0	2.146	.227	3.330	2.375
40.0	1.240	.115	2.280	1.150
45.0	.710	.058	1.594	.725

$$S \times 10^6 = .880 \quad S \times 10^6 = .1618$$

* Calculated assuming constant spectral density

** Calculated from results of iterative technique

Table 18

COMPARISON OF TOTAL POWERS OBTAINED

BY DIFFERENT TECHNIQUES

Run	(*) $P \times 10^6$ in ²	(2*) $P \times 10^6$ in ²	(3*) $P \times 10^6$ in ²	(4*) $P \times 10^6$ in ²	(2*) % diff.	(3*) % diff.	(4*) % diff.
1	13.4	15.1	16.3	14.1	+ 12.7	+ 21.6	+ 5.2
1A	13.4	22.9			+ 70.9		
1B	13.4	21.6			+ 60.4		
2	146.	94.6	102	95.1	- 35.2	- 23.2	- 34.9
3	523	713	731	783	+ 36.3	+ 39.8	+ 47.8
4	929	816	911	838	- 12.2	- 1.9	- 9.8
5	1570	1274	1368	1289	- 18.8	- 12.9	- 17.9
6	2480	2113	2170	2313	- 14.8	- 12.5	- 6.7
7	35.0	46.4	48.6	49.4	+ 32.6	+ 38.8	+ 41.2
8	552	652	645	581	+ 18.1	+ 16.8	+ 5.3
9	1270	1076	1066	974	- 15.3	- 16.1	- 23.3
10	1180	1168	1175	1092	- 1.0	- 0.4	- 7.5
11	132	133	134	135	+ 0.8	+ 1.5	+ 2.3
12	686	719	244	517	+ 4.8	- 64.4	- 24.6
13	3340	2190	2440	2550	- 34.4	- 27.0	- 23.6

* - Total power measured

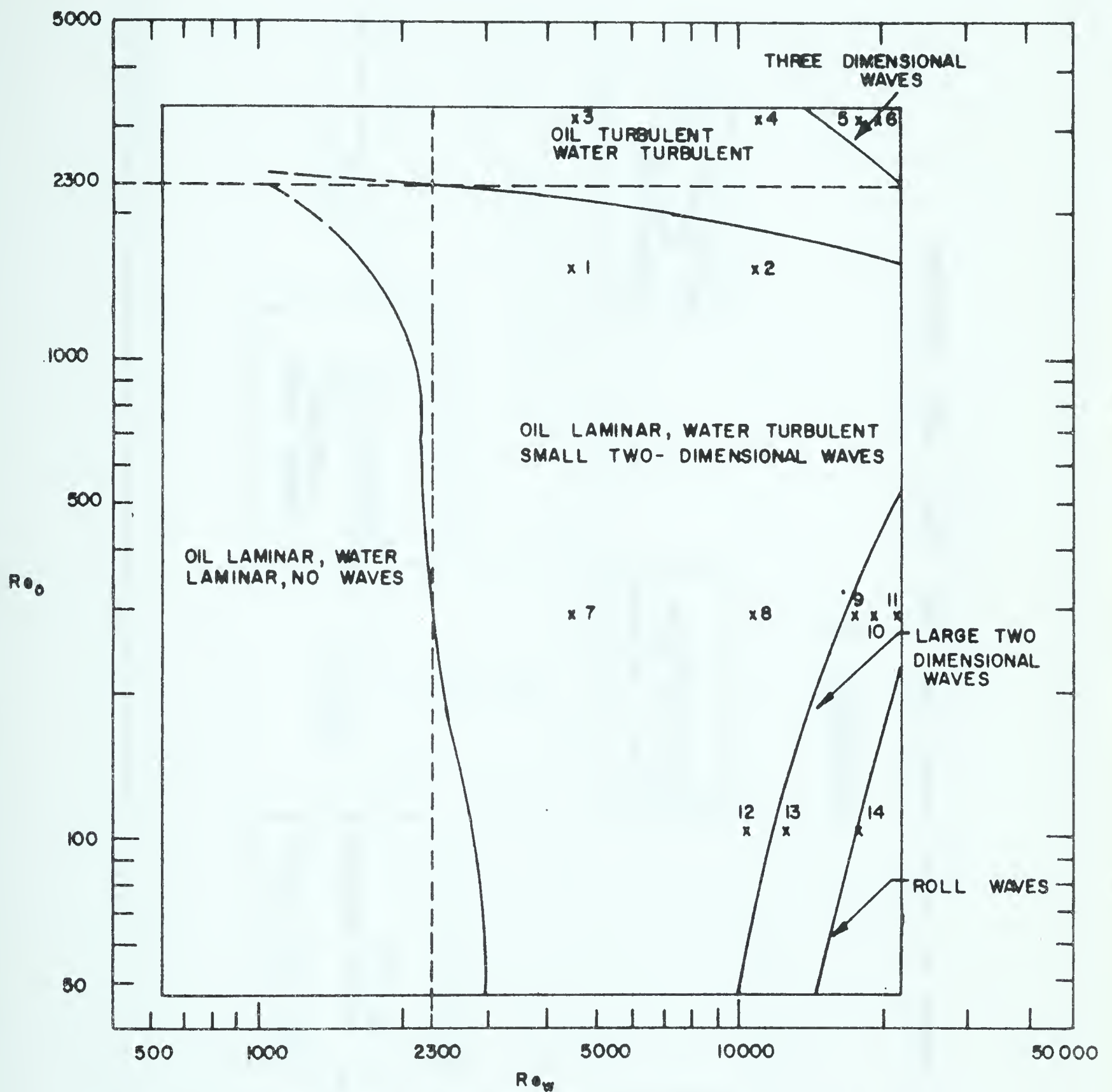
2* - Calculated at every value of frequency

3* - Calculated assuming constant density in the pass-band

4* - Calculated at every fifth value of frequency

Table 19
INTERFACE DISTRIBUTION

<u>Run</u>	<u>$N(\bar{h})$</u>	<u>$N(h_M)$</u>
1	15.8	10.2
1A		
1B		
2	31.8	22.2
3	34.8	32.8
4	31.2	29.2
5	40.2	33.8
6	37.8	31.4
7	16.3	12.0
8	29.2	28.4
9	32.2	34.2
10	36.7	35.1
11	18.1	10.0
12	16.2	26.8
13	34.6	35.7



x INDICATES POINTS WHERE RUNS WERE TAKEN (6)
 — INDICATES TRANSITION IN INTERFACIAL WAVE STRUCTURE

FIGURE 1

FLOW REGIMES AND INTERFACIAL STRUCTURE

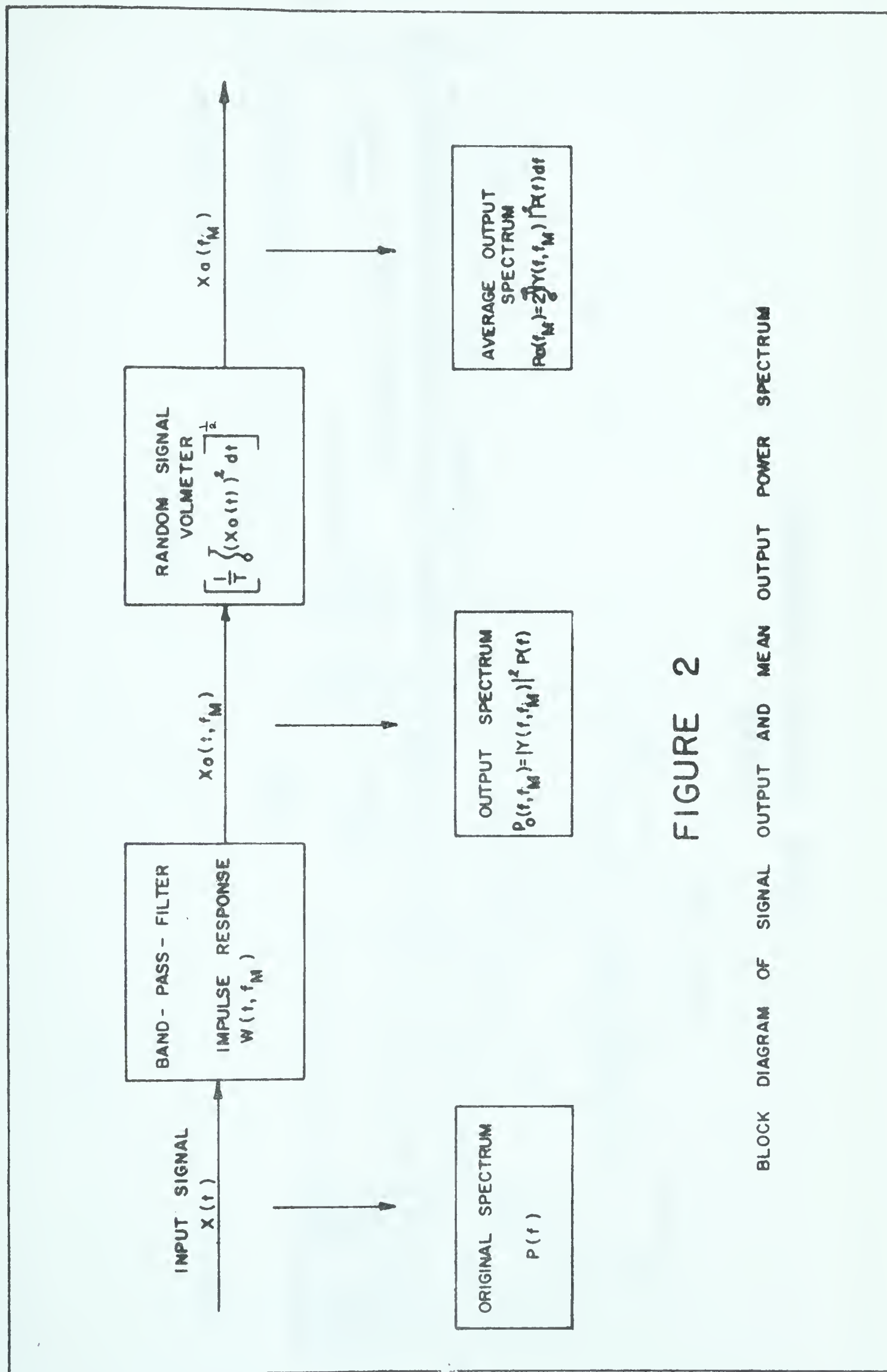


FIGURE 2

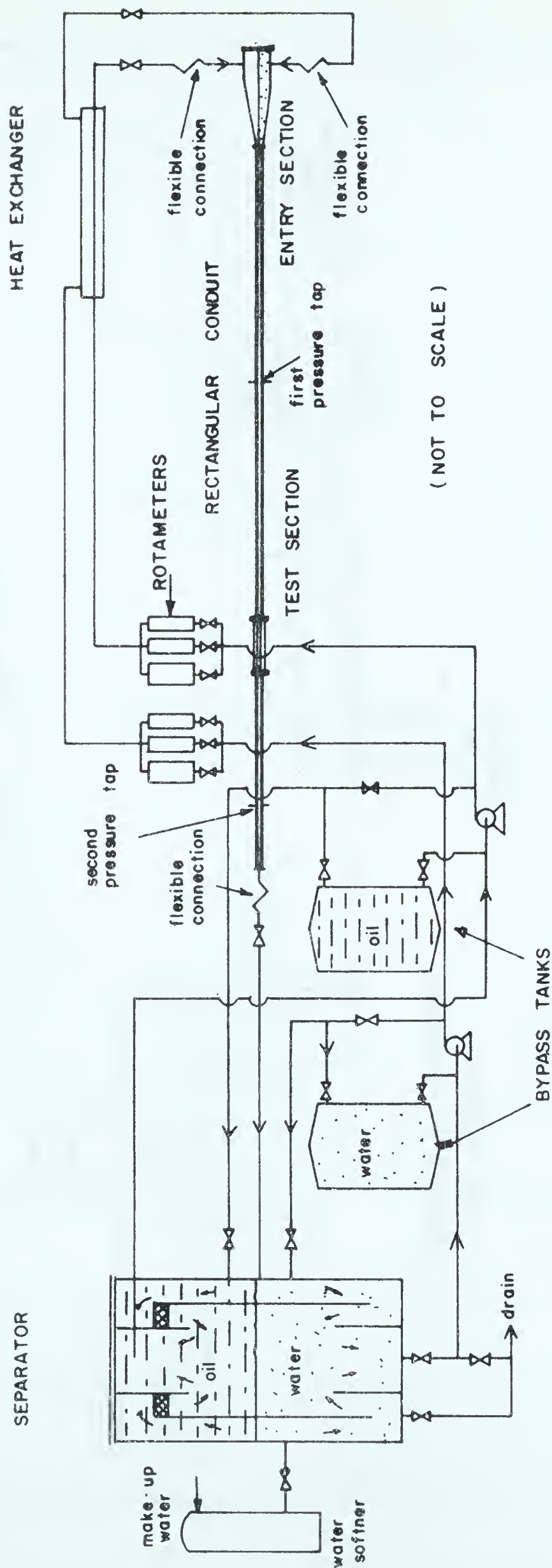


FIGURE 3
FLOW SYSTEM EQUIPMENT

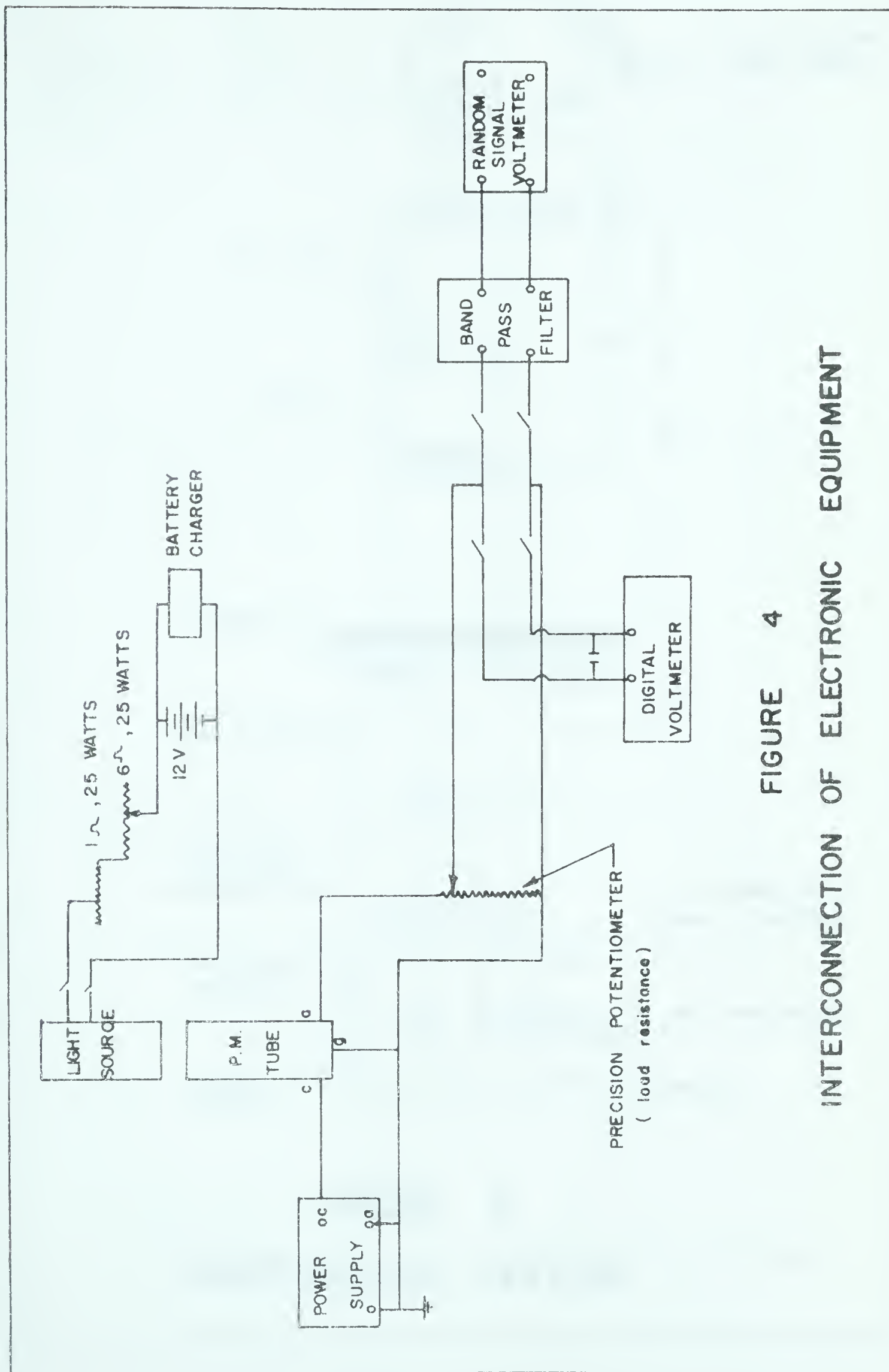


FIGURE 4
INTERCONNECTION OF ELECTRONIC EQUIPMENT

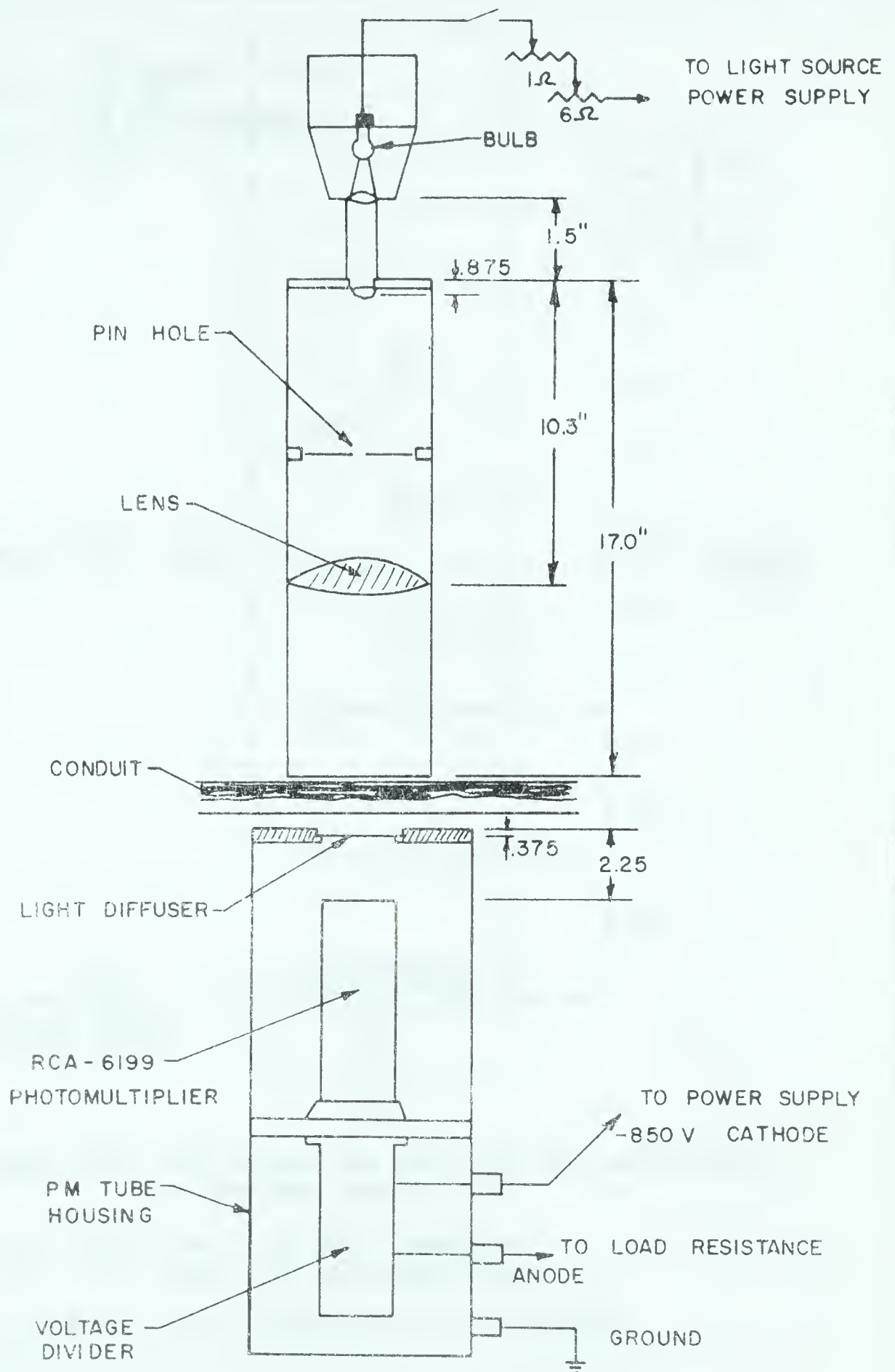
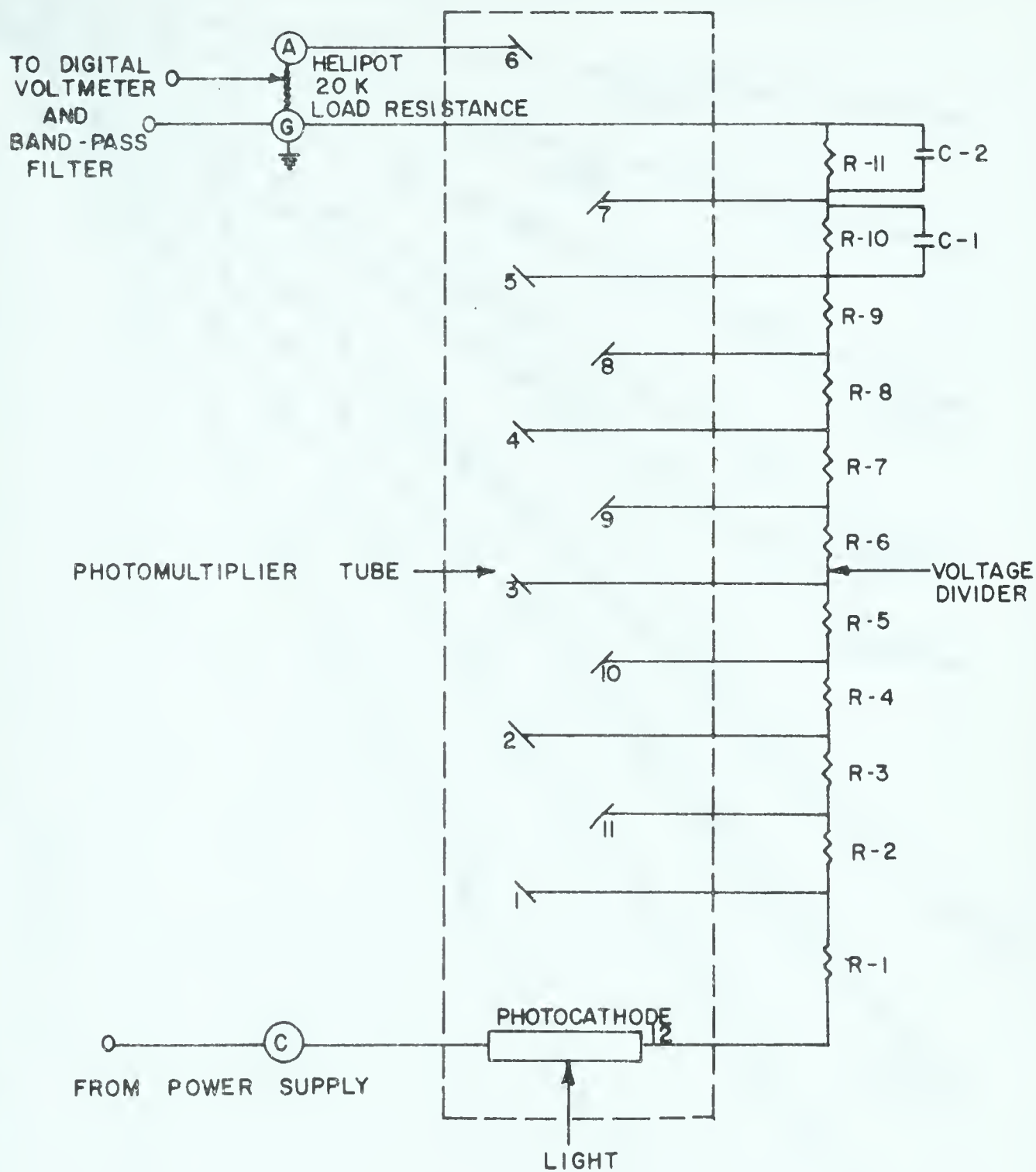


FIGURE 5

PHOTOMETRIC SYSTEM



NOS. 1 to 12 ARE DYNODE PIN NOS. GIVEN BY MANUFACTURER OF PHOTOMULTIPLIER TUBE

R-2 to R-11 = 47 K , 1/2 WATT RESISTORS

R-1 100K , 1/2 WATT RESISTORS

C-1 & C-2 = 8 μ f ELECTROLYTIC CAPACITORS

FIGURE 6

PHOTOMULTIPLIER TUBE CIRCUIT DIAGRAM

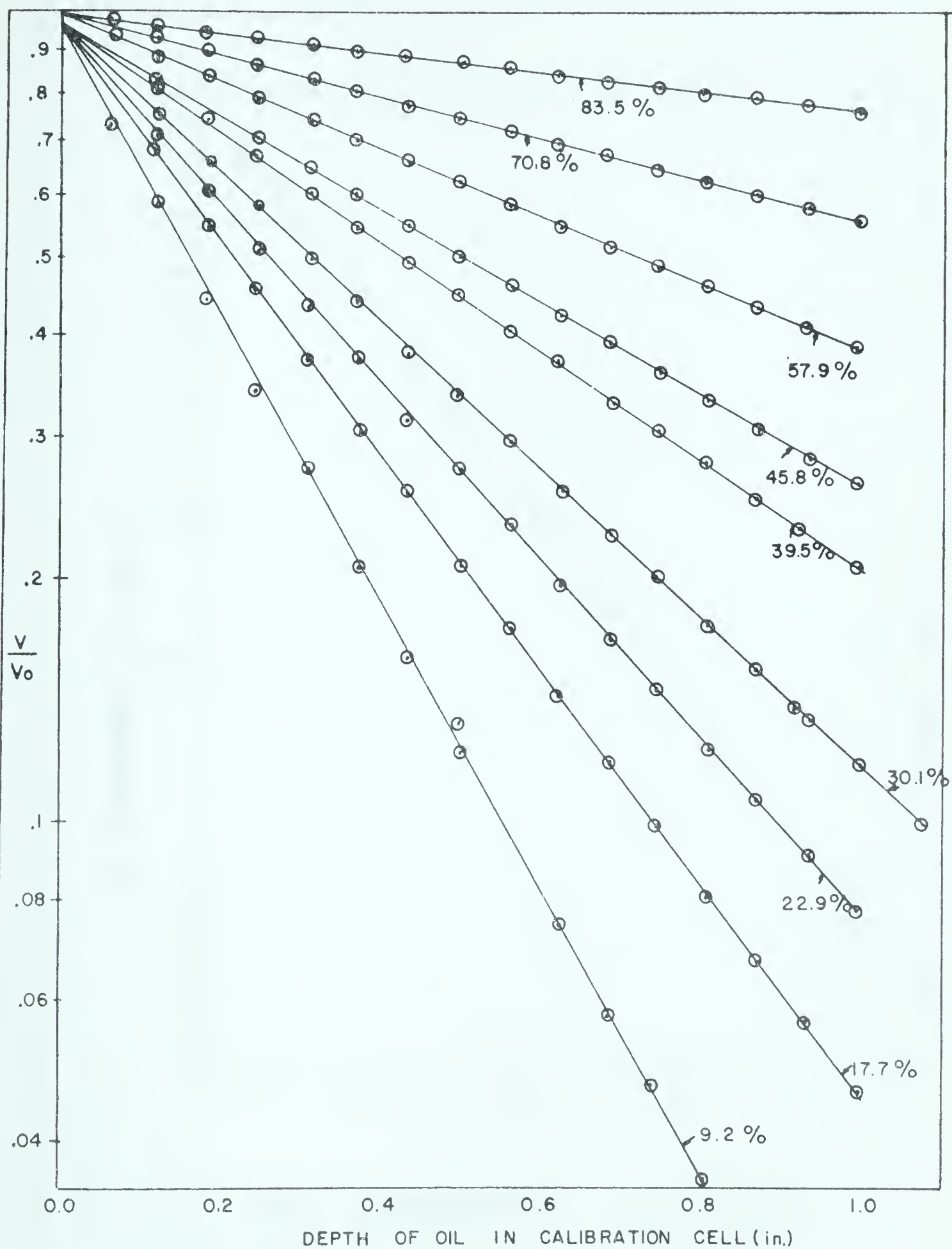
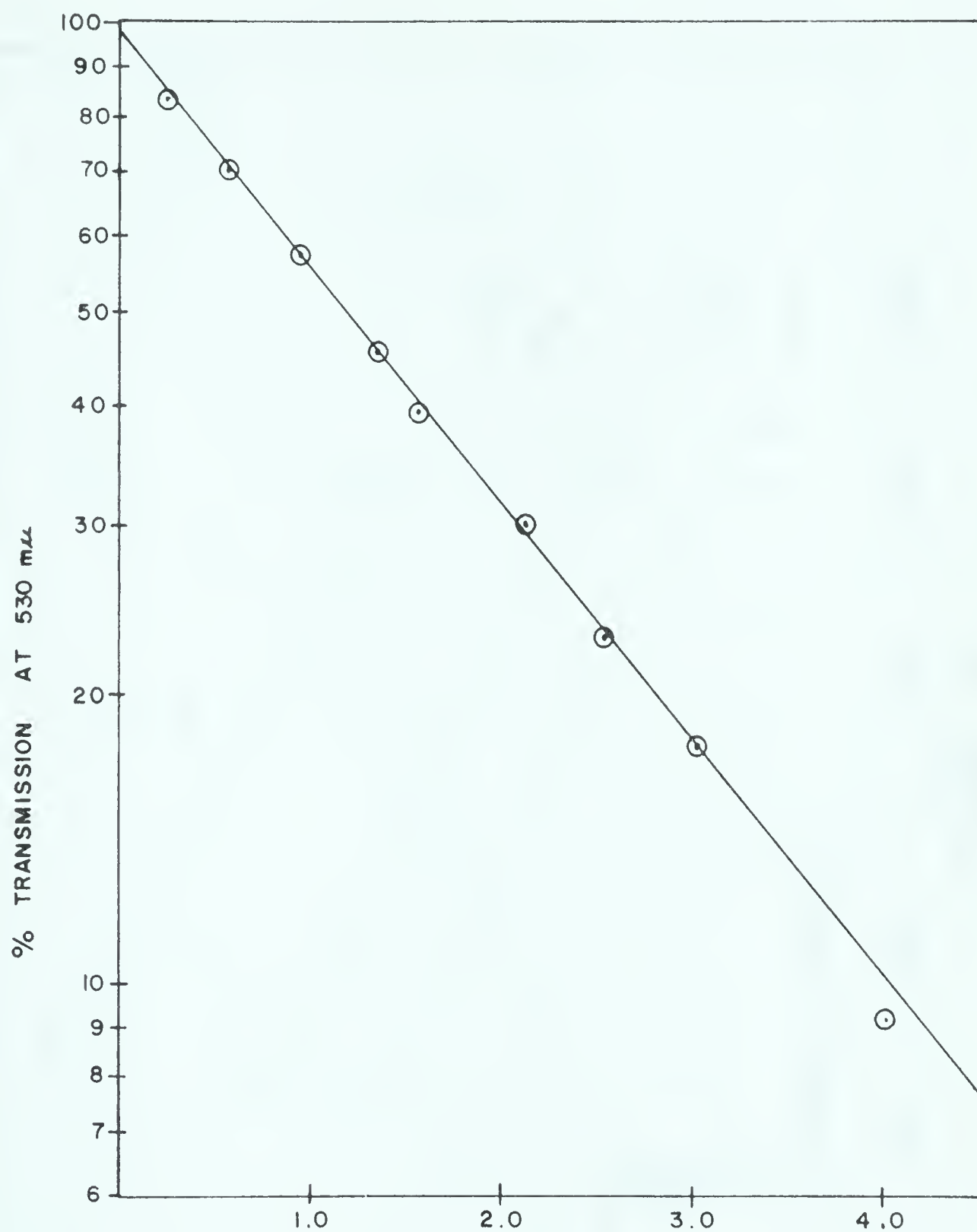


FIGURE 7

CALIBRATION OF DYE SOLUTIONS

WITH % TRANSMISSION AT 530 mμ AS THE PARAMETER



DYE SENSITIVITY- $K, \left(- \frac{\ln V/V_0}{\text{in. of } h} \right)$

FIGURE 8
DYE SOLUTION SENSITIVITY

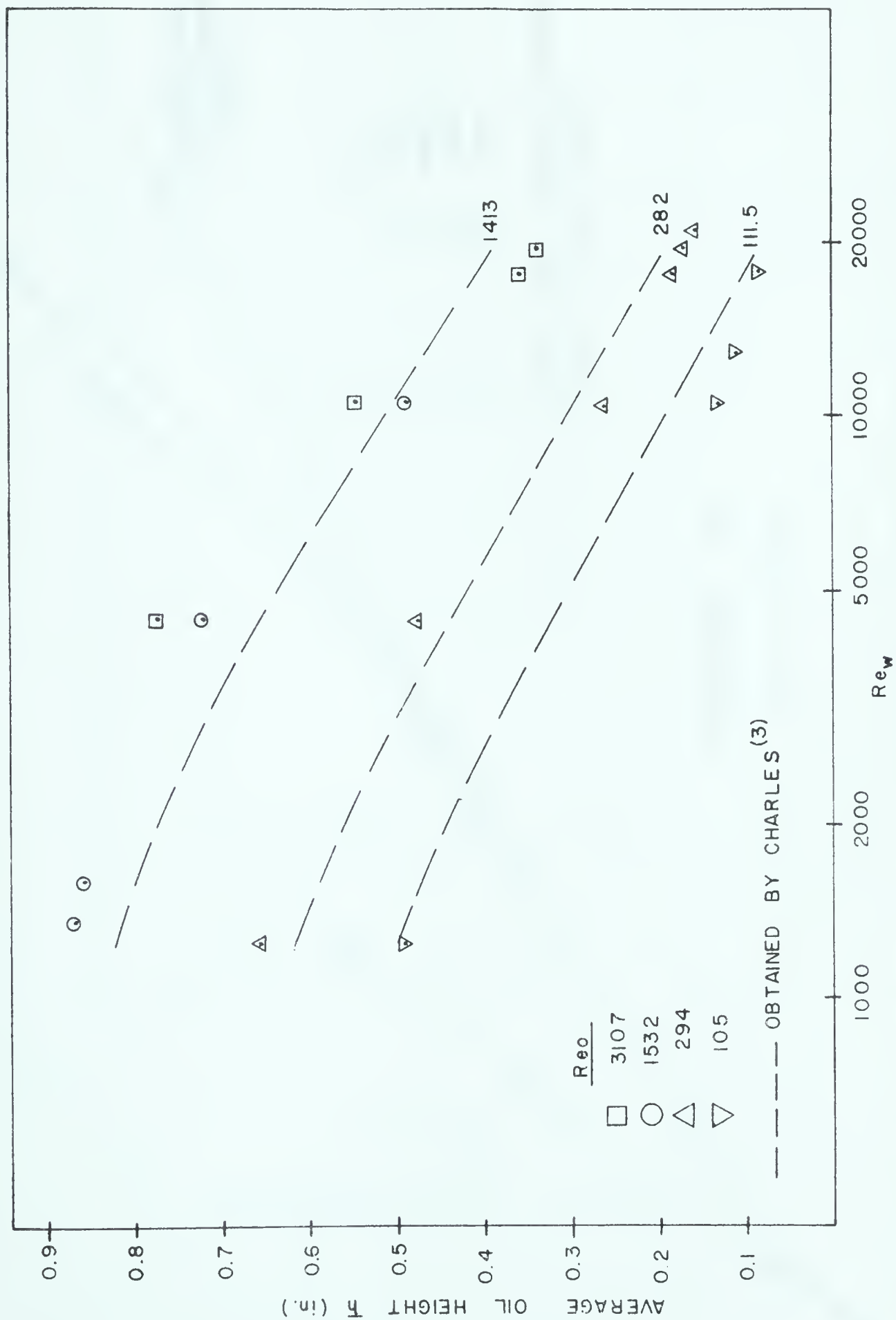
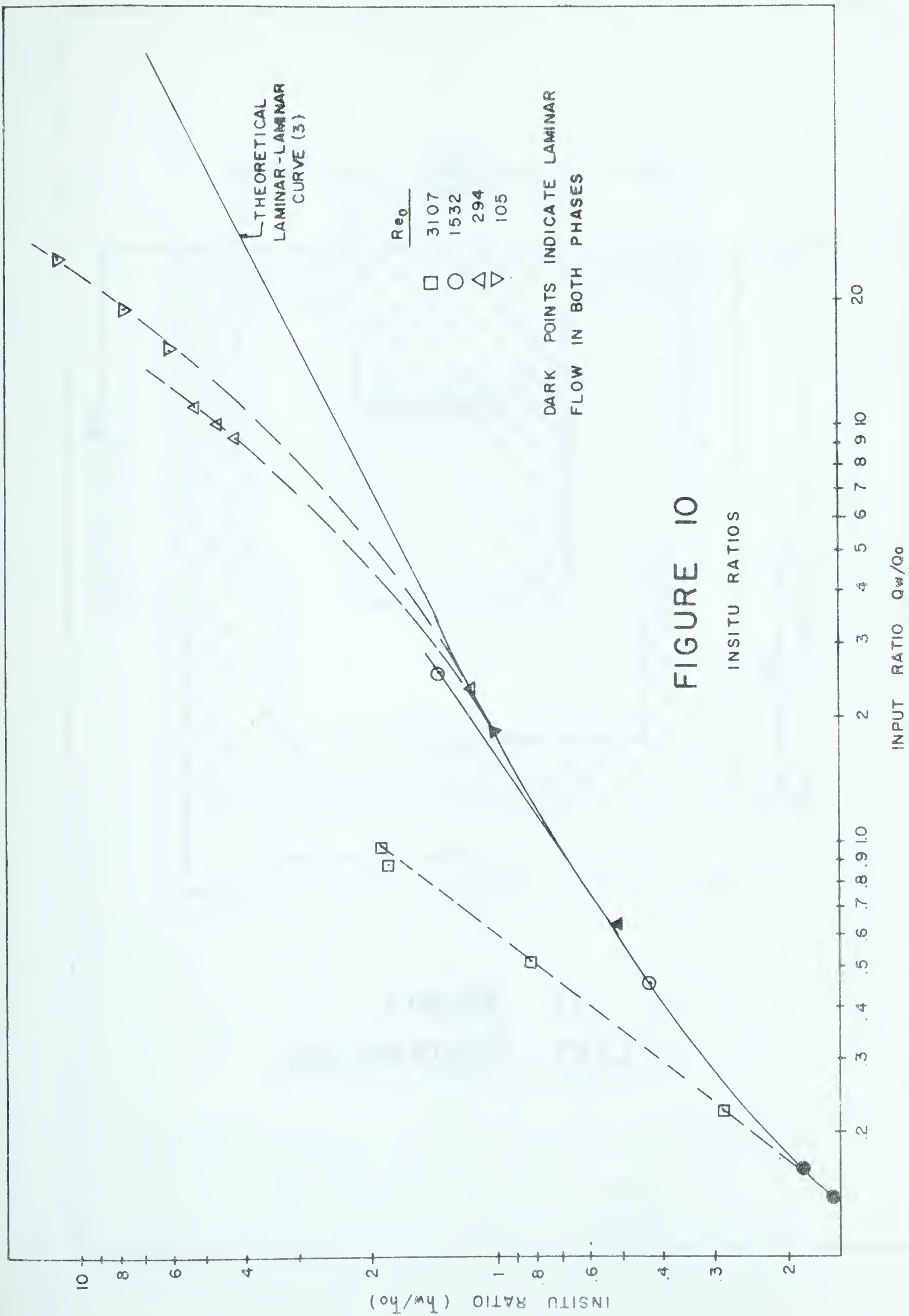


FIGURE 9
AVERAGE OIL FILM THICKNESS



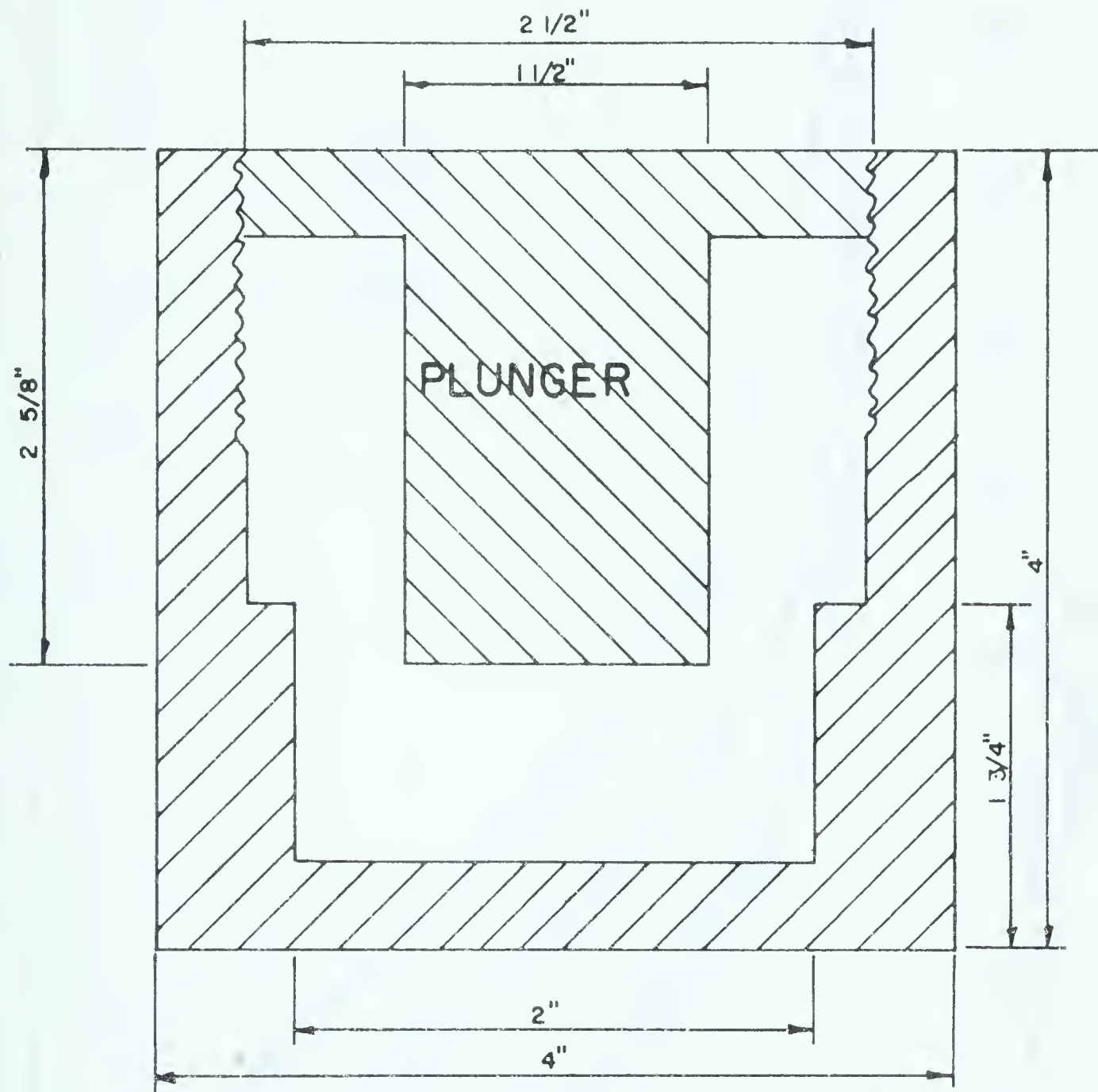


FIGURE II
CALIBRATION CELL

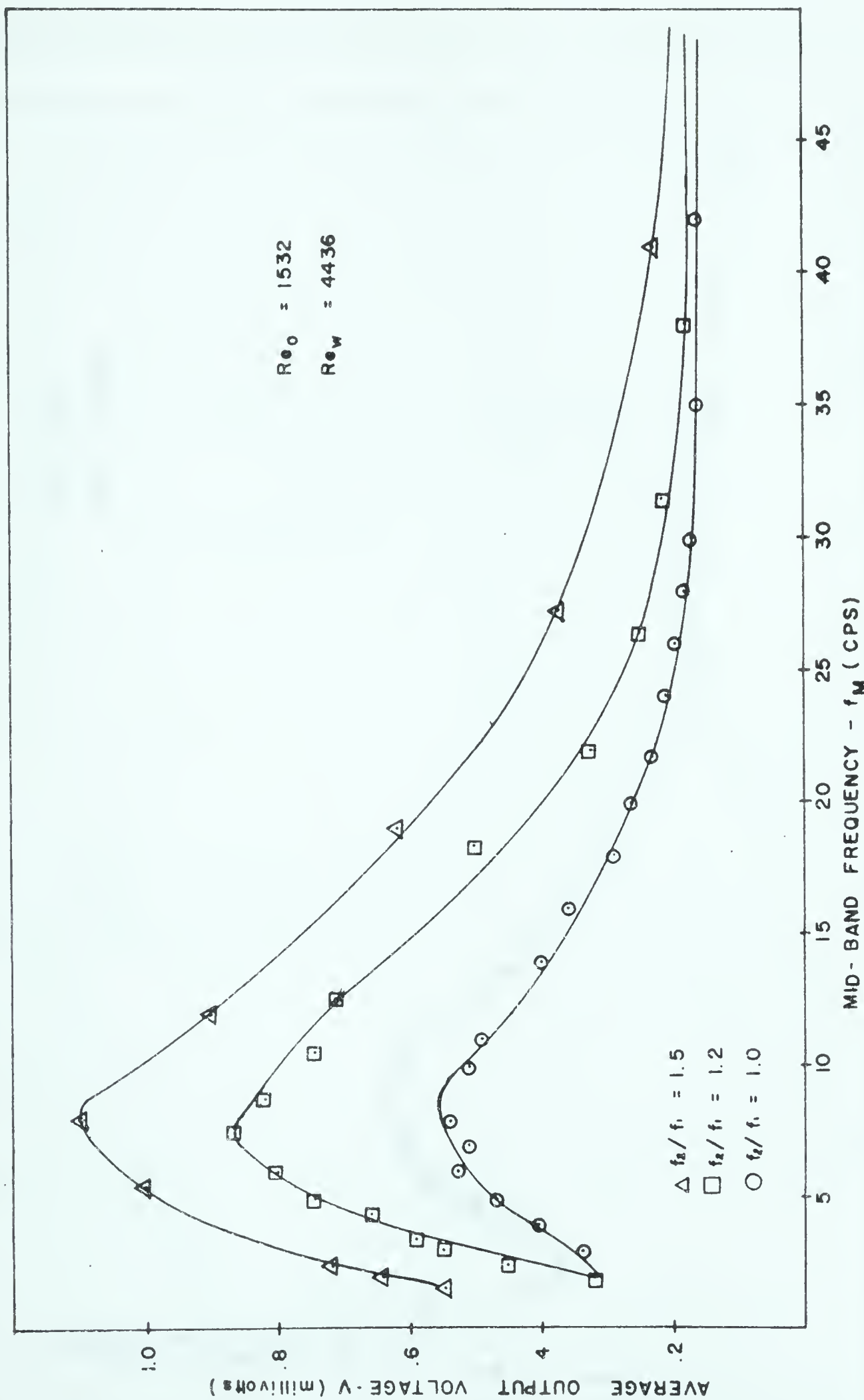


FIGURE 12

RAW DATA INDICATING EFFECT OF DIFFERENT PASS-BAND WIDTH

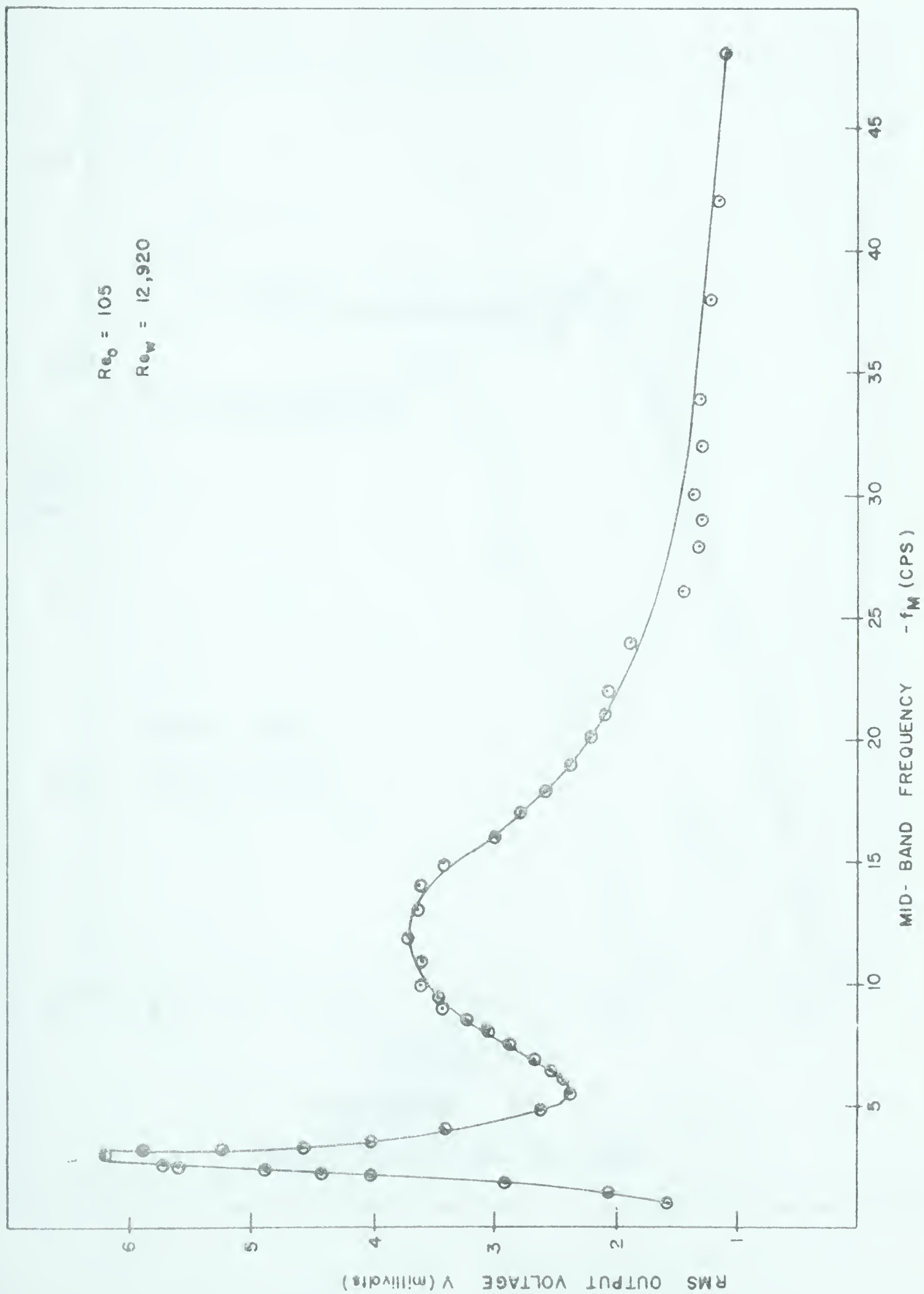


FIGURE 13

RAW DATA INDICATING PRESENCE OF TWO PEAKS

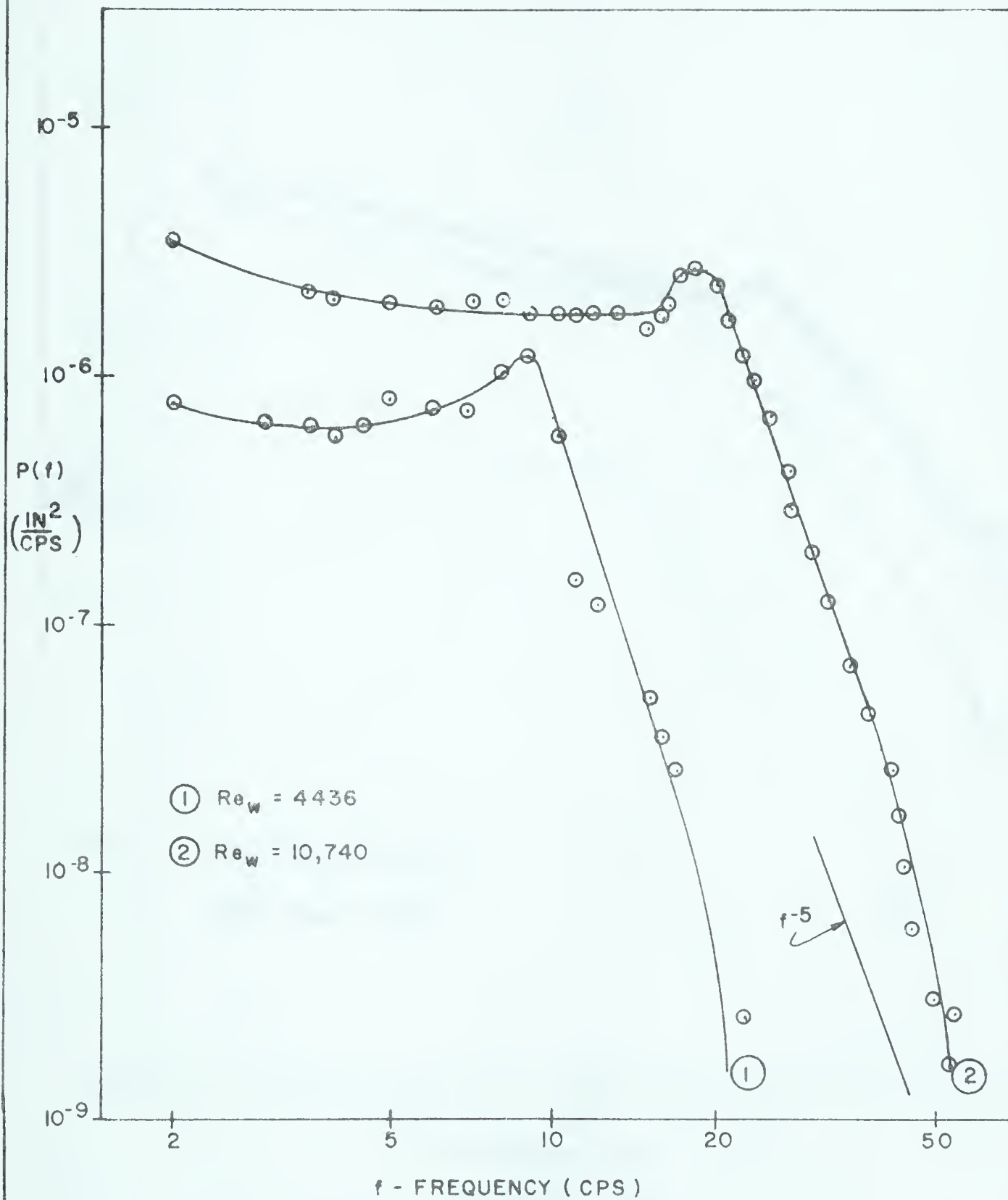
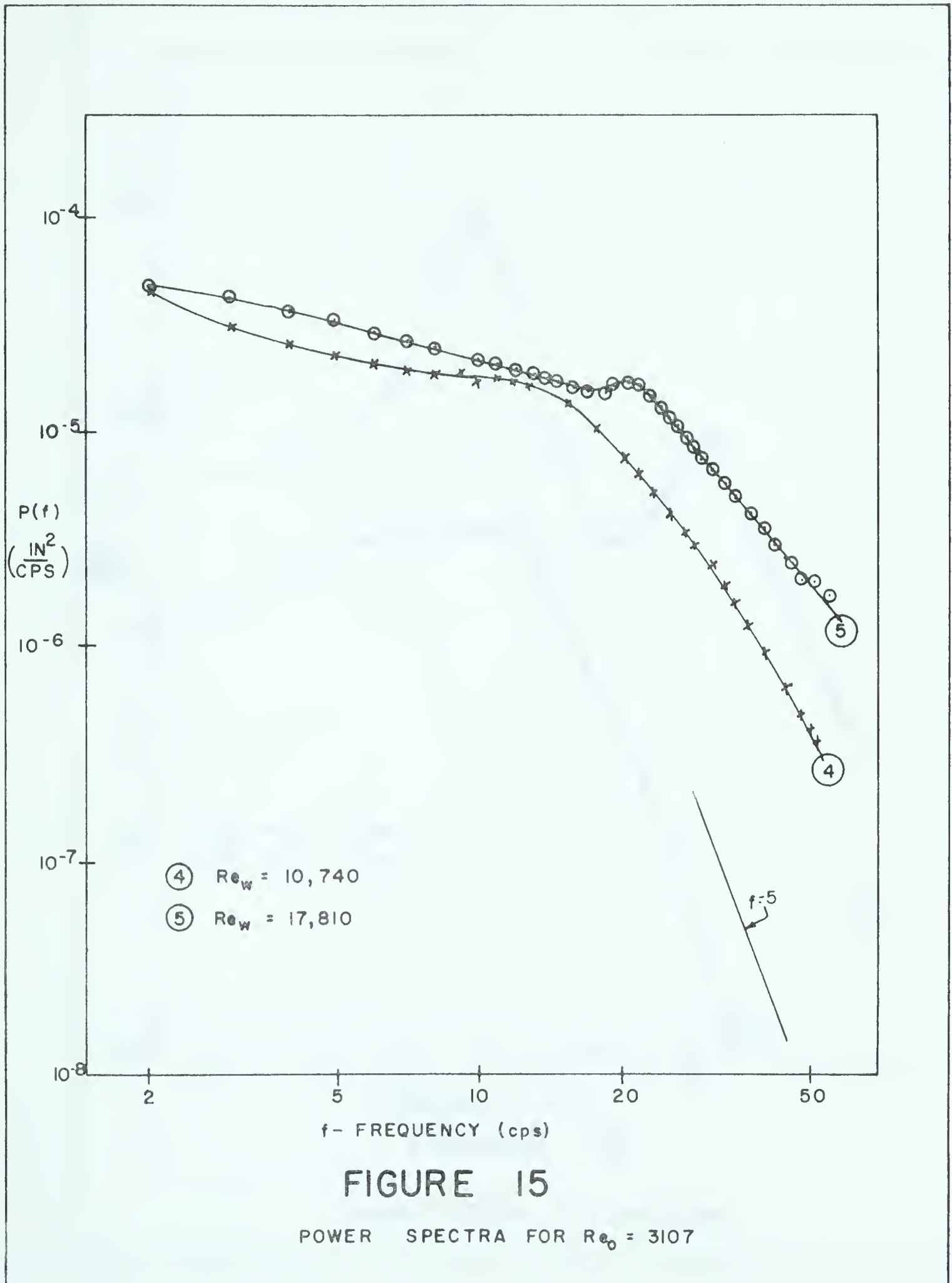
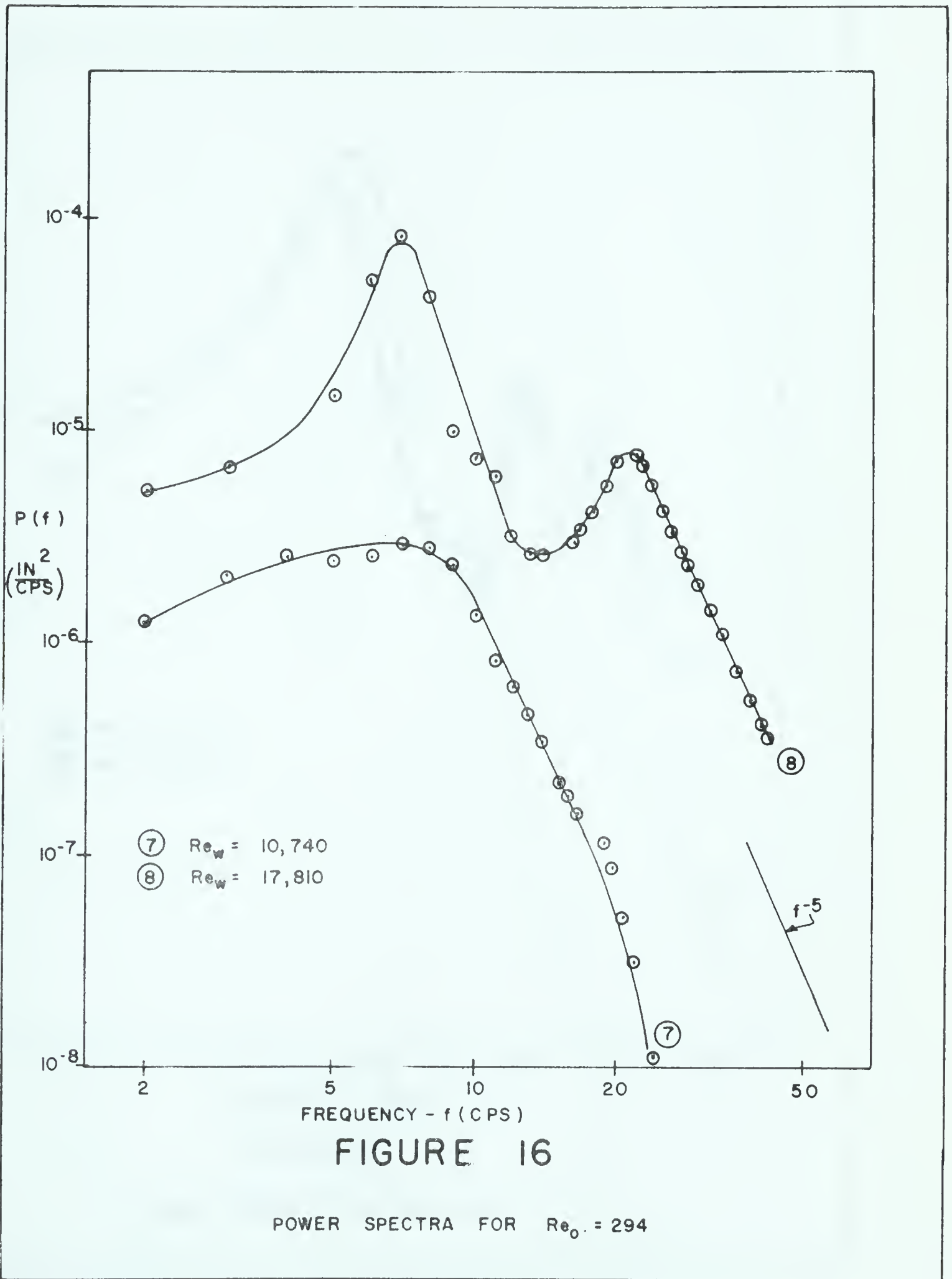


FIGURE 14

POWER SPECTRA FOR $Re_0 = 1532$





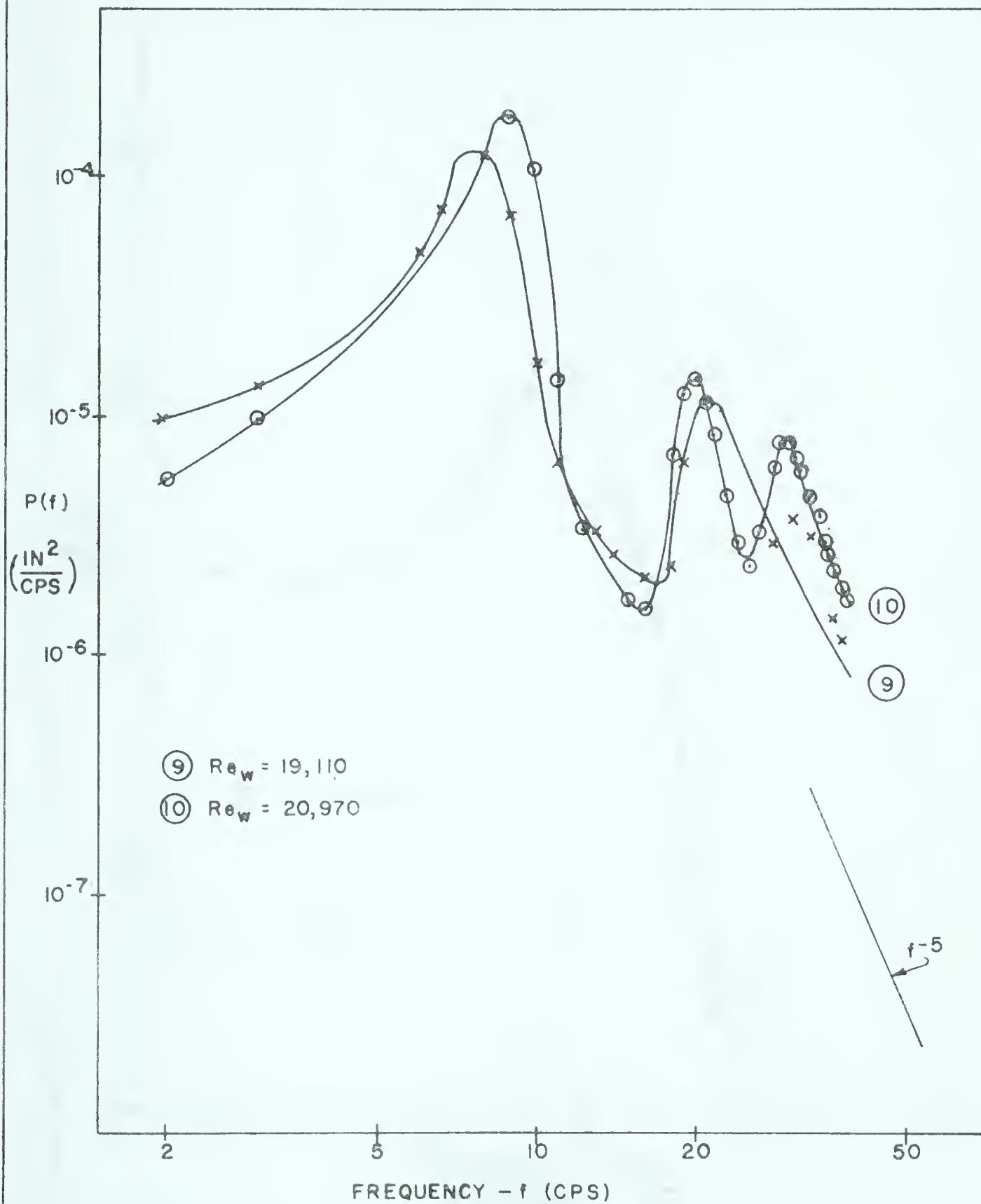
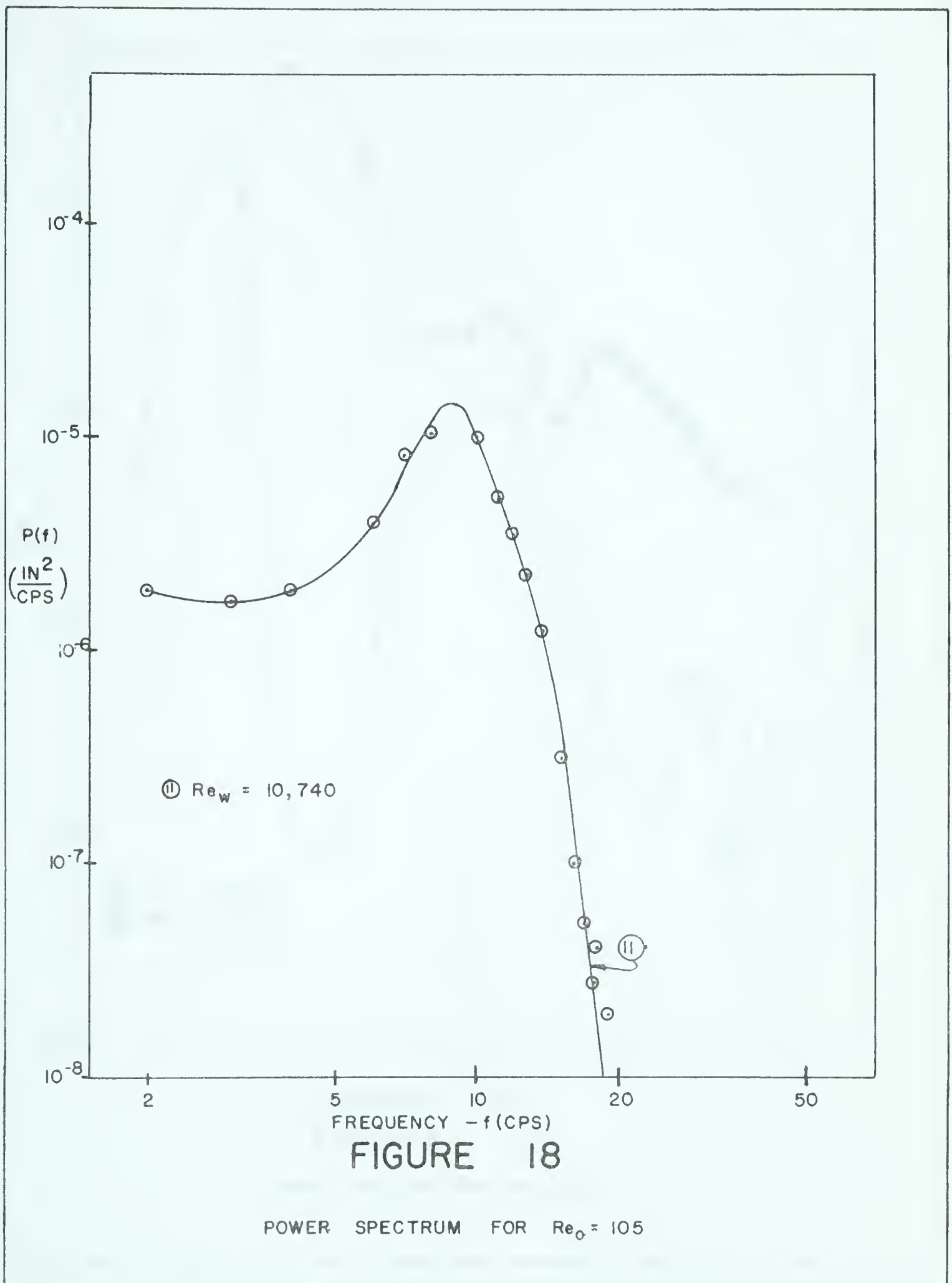
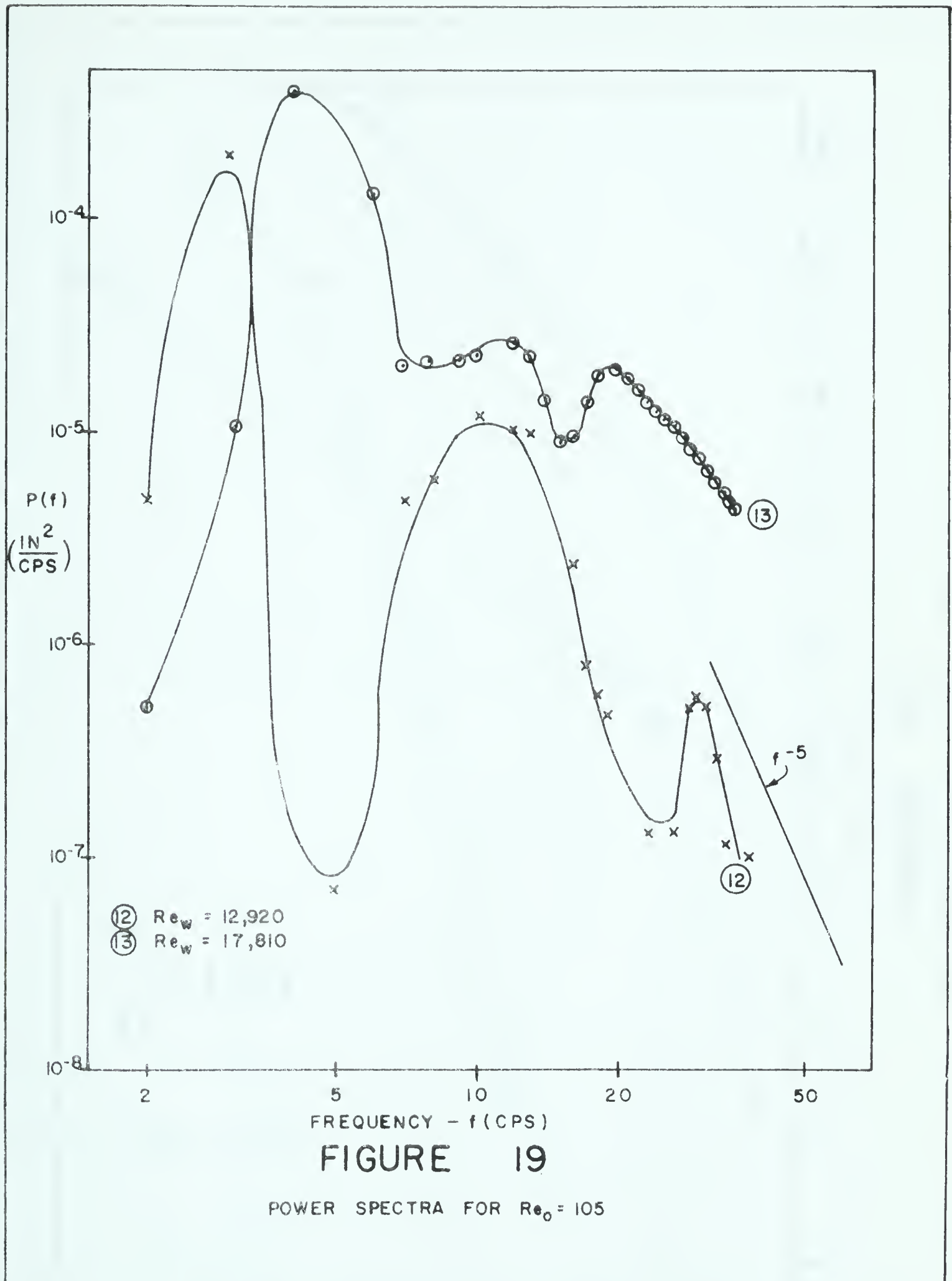


FIGURE 17

POWER SPECTRA FOR $Re_0 = 294$





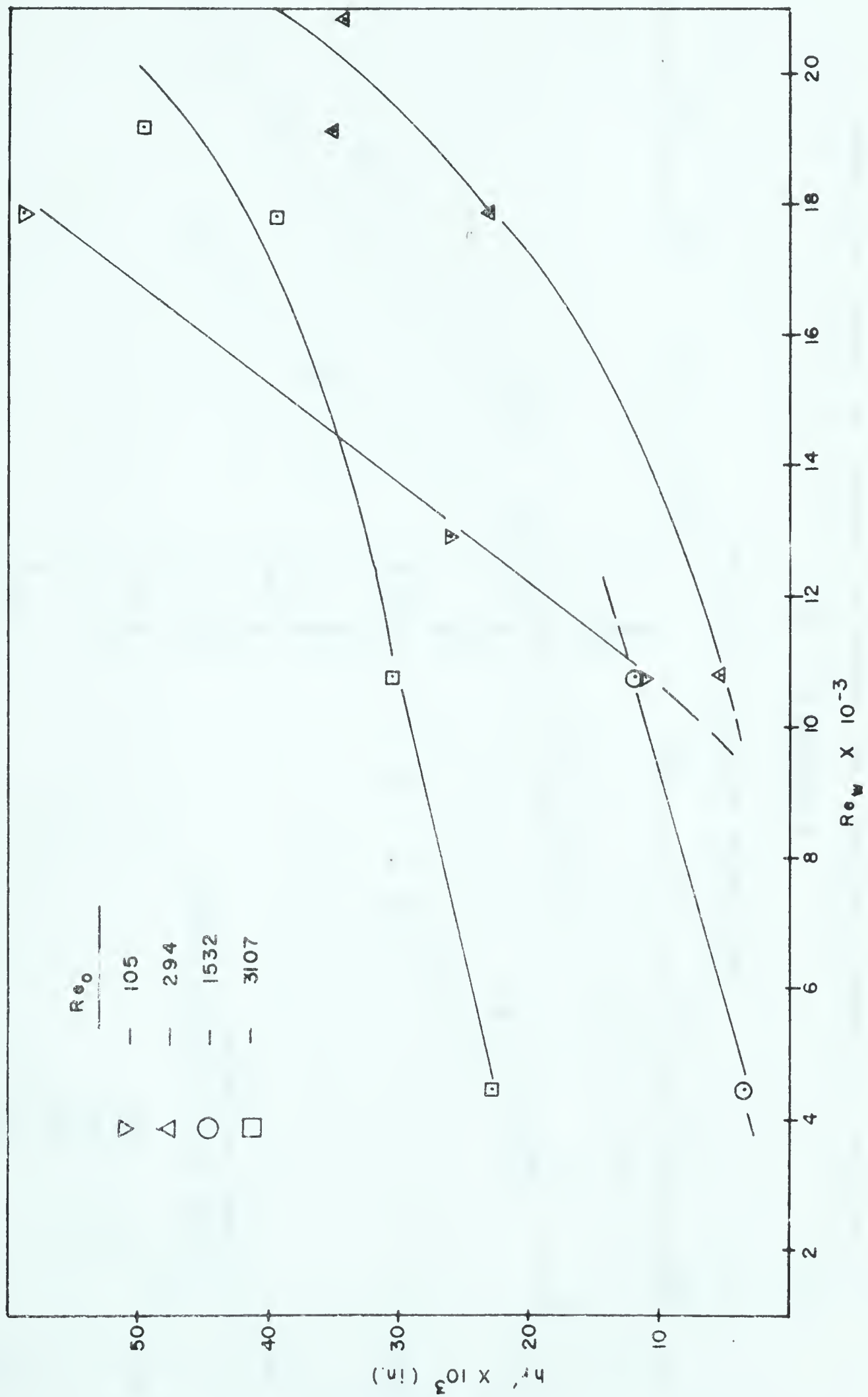


FIGURE 20

ROOT - MEAN - SQUARE SURFACE DISPLACEMENT

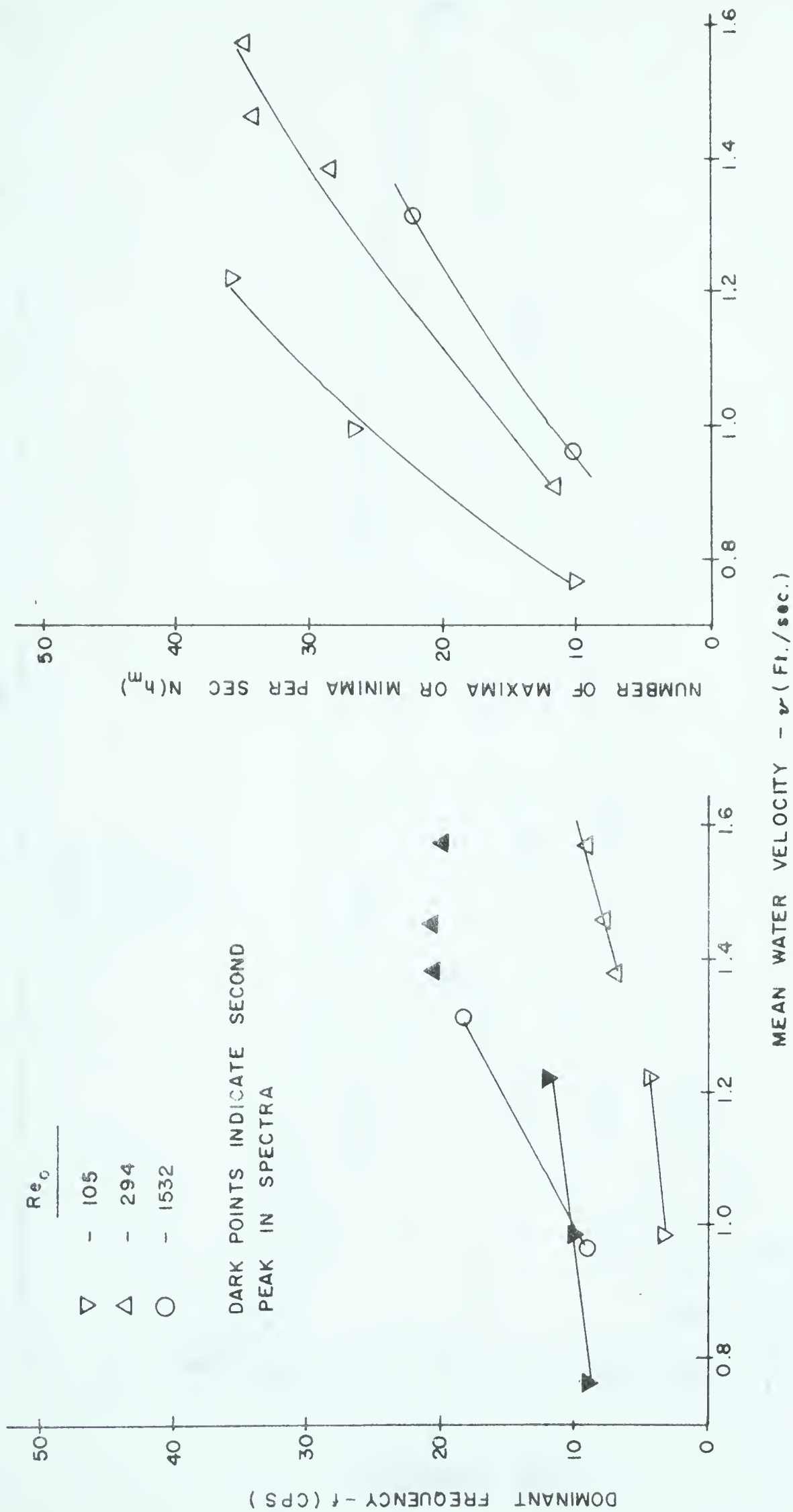


FIGURE 21

VARIATION OF DOMINANT FREQUENCY AND NUMBER OF MAXIMA OR MINIMA WITH MEAN WATER VELOCITY

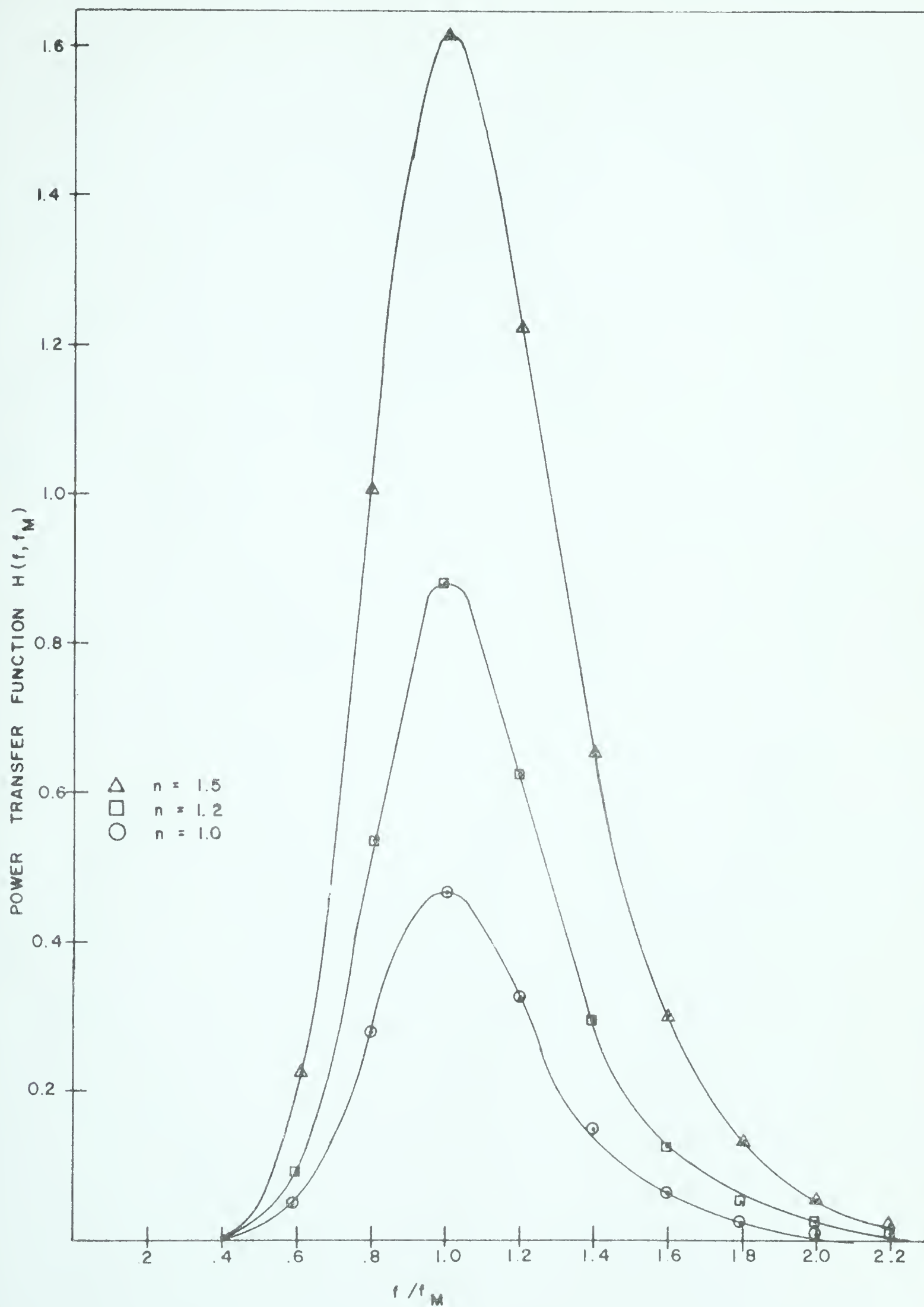


FIGURE 22

EFFECT OF PASS-BAND WIDTH ON POWER TRANSFER FUNCTION

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